

# Chapter 27: Methods of Differentiation

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## Outline

- Differentiation of Common Functions
- Differentiation of a Product
- Differentiation of a Quotient
- Function of a Function
- Successive Differentiation

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## Differentiation of Common Functions

- Summary of Standard Derivatives

$f(x)$	$f'(x)$
$ax^n$	$anx^{n-1}$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$
$e^{ax}$	$ae^{ax}$
$\ln ax$	$1/x$

- **Problem 4.** Find the derivatives of  
(a)  $y = 3\sqrt{x}$  (b)  $y = 5/\sqrt[3]{x^4}$   
[(a)  $3/2\sqrt{x}$  (b)  $-20/3\sqrt[3]{x^7}$ ]

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## Problems

- **Problem 6.** Find the differential coefficients of (a)  $y = 3 \sin 4x$  (b)  $f(t) = 2 \cos 3t$  with respect to the variable.  
[(a)  $12 \cos 4x$  (b)  $-6 \sin 3t$ ]
- **Problem 7.** Determine the derivatives of  
(a)  $y = 3e^{5x}$  (b)  $f(x) = 2/e^{3x}$  (c)  $y = 6 \ln 2x$ .  
[(a)  $15e^{5x}$  (b)  $-6/e^{3x}$  (c)  $6/x$ ]
- **Problem 9.** Determine the co-ordinates of the point on the graph  $y = 3x^2 - 7x + 2$  where the gradient is  $-1$ .  
[(1, -2)]

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## Exercise 117

Find the differential coefficients of the given functions with respect to the variable.

- **Exercise 4.** (a)  $-3/\sqrt[3]{x}$  (b)  $2 \sin 3x$

[(a)  $1/\sqrt[3]{x^4}$  (b)  $6 \cos 3x$ ]

- **Exercise 6.** (a)  $4 \ln 9x$  (b)  $\frac{e^x - e^{-x}}{2}$  (c)  $\frac{1 - \sqrt{x}}{x}$

[(a)  $\frac{4}{x}$  (b)  $\frac{e^x + e^{-x}}{2}$  (c)  $\frac{-1}{x^2} + \frac{1}{2\sqrt{x^3}}$ ]

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## Differentiation of a Product

- **Product rule:** When  $y = uv$ , and  $u$  and  $v$  are both functions of  $x$ , then

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

- **Problem 12.** Differentiate  $y = x^3 \cos 3x \ln x$ .  
[ $x^2\{\cos 3x + 3 \ln x (\cos 3x - x \sin 3x)\}$ ]

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## Exercise 118

- **Exercise 5.** Differentiate the given products with respect to the variable:  $e^t \ln t \cos t$

[ $e^t\{(1/t + \ln t)\cos t - \ln t \sin t\}$ ]

- **Exercise 7.** Evaluate  $dz/dt$ , correct to 4 significant figures, when  $t = 0.5$ , given that

$$z = 2e^{3t} \sin 2t.$$

[32.31]

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## Differentiation of a Quotient

- **Quotient rule:** When  $y = u/v$ , and  $u$  and  $v$  are both functions of  $x$ , then

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

- **Problem 14.** Find the differential coefficient of  $y = 4 \sin 5x / 5x^4$ .

[ $4(5x \cos 5x - 4 \sin 5x)/5x^5$ ]

- **Problem 15.** Determine the differential coefficient of  $y = \tan ax$ .

[ $a \sec^2 ax$ ]

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## Problems

- **Problem 16.** Find the derivative of  $y = \sec ax$ .  
[a sec ax tan ax]
- **Problem 17.** Differentiate  $y = te^{2t} / 2\cos t$ .  
[ $e^{2t}(2t \cos t + \cos t + t \sin t) / 2 \cos^2 t$ ]
- **Problem 18.** Determine the gradient of the curve  $y = 5x / (2x^2 + 4)$  at the point  $(\sqrt{3}, \sqrt{3}/2)$ .  
[-1/10]

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## Exercise 119

Differentiate quotients with respect to the variable.

- **Exercise 3.**  $\frac{3\sqrt{x^3}}{2\sin 2x}$   
[ $\frac{3\sqrt{x}(3\sin 2x - 4x\cos 2x)}{4\sin^2 2x}$ ]
- **Exercise 5.**  $\frac{2xe^{4x}}{\sin x}$   
[ $\frac{2e^{4x}}{\sin^2 x} \{(1 + 4x)\sin x - x\cos x\}$ ]

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## Function of a Function

- **Function of a function rule** or **chain rule**: If  $y$  is a function of  $x$ , then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

- **Problem 19.** Differentiate  $y = 3 \cos(5x^2 + 2)$ .  
[-30x sin(5x<sup>2</sup> + 2)]
- **Problem 20.** Find the derivative of  $y = (4t^3 - 3t)^6$ .  
[18(4t<sup>2</sup> - 1)(4t<sup>3</sup> - 3t)<sup>5</sup>]
- **Problem 22.** Differentiate  $y = 3 \tan^4 3x$ .  
[36 tan<sup>3</sup> 3x sec<sup>2</sup> 3x]

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## Exercise 120

Find the differential coefficients with respect to the variable.

- **Exercise 3.**  $2 \cos^5 x$   
[-10 cos<sup>4</sup> x sin x]
- **Exercise 6.**  $2 \cot(5t^2 + 3)$   
[-20t cosec<sup>2</sup>(5t<sup>2</sup> + 3)]

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## Successive Differentiation

- **Problem 24.** If  $f(x) = 2x^5 - 4x^3 + 3x - 5$ , find  $f''(x)$ .  
( $f''(x)$  pronounced f double-dash x)  
[ $4x(10x^2 - 6)$ ]
- **Problem 27.** Evaluate  $d^2y/dx^2$  when  $x = 0$  given  
 $y = 4 \sec 2x$ . ( $d^2y/dx^2$  pronounced dee two y by  
dee x squared)  
[ $d^2y/dx^2 = 16 \sec^3 2x + 16 \sec 2x \tan^2 2x, 16$ ]

## Exercise 121

In Exercise 3 and 4, find the second differential coefficient with respect to the variable.

- **Exercise 3.** (a)  $3 \sin 2t + \cos t$  (b)  $2 \ln 4x$   
[(a)  $-(12 \sin 2t + \cos t)$  (b)  $-2/x^2$  ]
- **Exercise 4.** (a)  $2 \cos^2 x$  (b)  $(2x - 3)^4$   
[(a)  $4(\sin^2 x - \cos^2 x)$  (b)  $48(2x - 3)^2$ ]