

Chapter 3: Partial Fractions

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Outline

- Introduction to Partial Fractions
- Linear Factors
- Repeated Linear Factors
- Quadratic Factors

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Introduction to Partial Fractions (1/2)

- By algebraic addition,

$$\frac{1}{x-2} + \frac{3}{x+1} = \frac{(x+1)+3(x-2)}{(x-2)(x+1)} = \frac{4x-5}{x^2-x-2}$$

The reverse process of moving from $\frac{4x-5}{x^2-x-2}$

to $\frac{1}{x-2} + \frac{3}{x+1}$ is called resolving into **partial**

fractions.

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Introduction to Partial Fractions (2/2)

- In order to resolve an algebraic expression into partial fractions:
 1. The **denominator must factorize** and
 2. The **numerator must be at least one degree less than the denominator**

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Three Types of Partial Fractions

Type	Denominator containing	Expression	Form of partial function
1	Linear factors (see Problems 1 to 4)	$\frac{f(x)}{(x+a)(x-b)(x+c)}$	$\frac{A}{(x+a)} + \frac{B}{(x-b)} + \frac{C}{(x+c)}$
2	Repeated linear factors (see Problems 5 to 7)	$\frac{f(x)}{(x+a)^3}$	$\frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3}$
3	Quadratic factors (see Problems 8 to 9)	$\frac{f(x)}{(ax^2+bx+c)(x+d)}$	$\frac{Ax+B}{(ax^2+bx+c)} + \frac{C}{(x+d)}$

- $f(x)$ is assumed to be of less degree than the relevant denominator
- A , B , and C are constant to be determined.

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Linear Factors (1/2)

- **Problem 1.** Resolve $(11 - 3x)/(x^2 + 2x - 3)$ into partial fractions.
[To determine constants A and B , values of x are chosen to make the term in A or B equal to zero.]
[$2/(x - 1) - 5/(x + 3)$]
- **Problem 2.** Convert $\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$ into the sum of three partial fractions.
[$4/(x + 1) - 3/(x - 2) + 1/(x + 3)$]

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Linear Factors (2/2)

- **Problem 4.** Express $\frac{x^3 - 2x^2 - 4x - 4}{x^2 + x - 2}$ in partial fractions.
[$x - 3 + 4/(x + 2) - 3/(x - 1)$]

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Exercise 13

- Resolve the following into partial fractions.
- **Exercise 3.** $(x^2 - 3x + 6)/[x(x - 2)(x - 1)]$
[$3/x + 2/(x - 2) - 4/(x - 1)$]
 - **Exercise 5.** $(x^2 + 9x + 8)/(x^2 + x - 6)$
[$1 + 2/(x + 3) + 6/(x - 2)$]

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Repeated Linear Factors

- **Problem 6.** Express $\frac{5x^2 - 2x - 19}{(x + 3)(x - 1)^2}$ as the sum of three partial fractions.
[$\frac{2}{x + 3} + \frac{3}{x - 1} - \frac{4}{(x - 1)^2}$]
- **Problem 7.** Resolve $(3x^2 + 16x + 15)/(x + 3)^3$ into partial fractions.
[$\frac{3}{x + 3} - \frac{2}{(x + 3)^2} - \frac{6}{(x + 3)^3}$]

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Exercise 14

- **Exercise 2.** $(x^2 + 7x + 3)/[x^2(x + 3)]$
[$\frac{1}{x^2} + \frac{2}{x} - \frac{1}{x + 3}$]
- **Exercise 3.** $(5x^2 - 30x + 44)/(x - 2)^3$
[$\frac{5}{x - 2} - \frac{10}{(x - 2)^2} + \frac{4}{(x - 2)^3}$]

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Quadratic Factors & Exercise 15

- **Problem 8.** Express $\frac{7x^2 + 5x + 13}{(x^2 + 2)(x + 1)}$ in partial fractions.
[$\frac{2x + 3}{x^2 + 2} + \frac{5}{x + 1}$]
- **Exercise 3.** $(15 + 5x + 5x^2 - 4x^3)/[x^2(x^2 + 5)]$
[$\frac{1}{x} + \frac{3}{x^2} + \frac{2 - 5x}{x^2 + 5}$]

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