Chapter 47: Homogeneous First Order Differential Equations

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Introduction

- Certain first order differential equations are not of the 'variable-separable' type, but can be made separable by changing the variable.
- An equation of the form $P\frac{\mathrm{d}y}{\mathrm{d}x}=Q$, where P and Q are functions of both x and y of the same degree throughout, is said to be homogeneous in y and x.

Procedure

- Rearrange $P\frac{\mathrm{d}y}{\mathrm{d}x}=Q$ into the form $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{Q}{P}$
- Make the substitution $y = vx$ (where v is a function of x), from which, $\frac{\mathrm{d}y}{\mathrm{d}x}=v(1)+x\frac{\mathrm{d}v}{\mathrm{d}x}$, by the product rule.
- Substitute for both y and $\frac{\mathrm{d}y}{\mathrm{d}x}$ in the equation $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{Q}{P}$. Simplify, by canceling, and an equation results in which the variables are separable.
- Separate the variables and solve.
- Substitute $v=\frac{y}{x}$ to solve in terms of the original variables.

Worked Problems

- **Problem 1.** Solve the differential equation: $y - x = x\frac{dy}{dx}$, given $x = 1$ when $y = 2$.
  [y = -x(ln x - 2)]

- **Problem 2.** Find the particular solution of the equation: $x\frac{dy}{dx} = \frac{x^2 + y^2}{y}$, given the boundary conditions that $y = 4$ when $x = 1$.
  [$y^2 = 2x^2(8 + \ln x)$]

Exercise 181

- **Exercise 3.** Find the particular solution of the differential equation: $(x^2 + y^2)dy = xy\,dx$, given that $x = 1$ when $y = 1$.
  [$x^2 = 2y^2(\ln y + 1/2)$]

- **Exercise 4.** Solve the differential equation:
  $$\frac{x+y}{y-x} = \frac{dy}{dx}$$
  [$x^2 + 2xy - y^2 = k$]

Further Worked Problems & Exercise 181

- **Problem 3.** Solve the equation: $7x(x - y)dy = 2(x^2 + 6xy - 5y^2)dx$ given that $x = 1$ when $y = 0$.
  [$\left(\frac{x+3y}{x}\right)^{4/3}\left(\frac{2x-y}{x}\right) = 2x$]

- **Exercise 2.** Solve: $(9xy - 11xy)\frac{dy}{dx} = 11y^2 - 16xy + 3x^2$
  [$\frac{1}{5}\left\{\frac{3}{13}\ln\left(\frac{13y-3x}{x}\right) - \ln\left(\frac{y-x}{x}\right)\right\} = \ln x + c$]

Exercise 182

- **Exercise 3.** Solve the differential equation: $2x\frac{dy}{dx} = x + 3y$, given that when x = 1, y = 1.
  [$(x + y)^2 = 4x^3$]

- **Exercise 4.** Show that the solution of the differential equation: $2xy\frac{dy}{dx} = x^2 + y^2$ can be expressed as: $x = K(x^2 - y^2)$, which K is a constant.