

Chapter 43: Integration by Parts

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Outline

- Introduction
- Worked Problems on Integration by Parts
- Further Worked Problems on Integration by Parts

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Introduction (1/2)

- From the product rule of differentiation:

$$\frac{d}{dx}(uv) = v \frac{du}{dx} + u \frac{dv}{dx},$$

where u and v are both functions of x .

- Rearranging gives: $u \frac{dv}{dx} = \frac{d}{dx}(uv) - v \frac{du}{dx}$
- Integrating both sides with respect to x gives:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

or $\int u dv = uv - \int v du$

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Introduction (2/2)

- This is known as the **integration by parts formula** and provides a method of integrating such products of simple functions as $\int xe^x dx$, $\int t \sin t dt$, $\int e^\theta \cos \theta d\theta$, and $\int x \ln x dx$.
- The choice must be such that
 - the 'u part' **becomes a constant after successive differentiation**, and
 - the 'dv part' **can be integrated from standard integrals**.

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Worked Problems

- **Problem 1.** Determine $\int x \cos x \, dx$.
[$x \sin x + \cos x + c$]
- **Problem 4.** Evaluate $\int_0^1 5xe^{4x} \, dx$, correct to 3 significant figures.
[51.5]
- **Problem 5.** Determine $\int x^2 \sin x \, dx$.
[$(2 - x^2)\cos x + 2x \sin x + c$]

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Exercise 168

- **Exercise 5.** Determine the integral using integration by parts: $\int 3t^2 e^{2t} \, dt$
[$3e^{2t}(t^2 - t + \frac{1}{2})/2 + c$]
- **Exercise 8.** Evaluate the integral, correct to 4 significant figures: $\int_0^{\frac{\pi}{2}} t^2 \cos t \, dt$
[0.4674]

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Further Worked Problems

- **Problem 6.** Find $\int x \ln x \, dx$.
[$x^2(2 \ln x - 1)/4 + c$]
- **Problem 7.** Determine $\int \ln x \, dx$.
[$x(\ln x - 1) + c$]
- **Problem 9.** Find $\int e^{ax} \cos bx \, dx$.
[$\frac{e^{ax}}{a^2 + b^2} (b \sin bx + a \cos bx) + c$]

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Exercise 169

Determine the integrals using integration by parts:

- **Exercise 3.** $\int x^2 \sin 3x \, dx$
[$\frac{\cos 3x}{27} (2 - 9x^2) + \frac{2}{9} x \sin 3x + c$]
- **Exercise 4.** $\int 2e^{5x} \cos 2x \, dx$
[$\frac{2}{29} e^{5x} (2 \sin 2x + 5 \cos 2x) + c$]

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