

Chapter 39: Integration Using Algebraic Substitutions

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2008 Spring

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Outline

- Algebraic Substitutions
- Worked Problems on Integration Using Algebraic Substitutions
- Further Worked Problems on Integration Using Algebraic Substitutions
- Change of Limits

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Algebraic Substitutions

- With **algebraic substitutions**, the substitution usually made is to let u be equal to $f(x)$ such that $f(u)du$ is a standard integral.
- It is found that integrals of the forms,

$$k \int [f(x)]^n f'(x) dx \text{ and } k \int \frac{f'(x)}{[f(x)]^n} dx$$

(where k and n are constants) can both be integrated by substituting u for $f(x)$.

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Worked Problems on Integration Using Algebraic Substitutions

- **Problem 2.** Find $\int (2x-5)^7 dx$.
(Note: expand $(2x-5)^7$ would be a lengthy process)
[$(2x-5)^8/16 + c$]
- **Problem 5.** Determine $\int 3x(4x^2+3)^5 dx$.
[$(4x^2+3)^6/16 + c$]
- **Problem 6.** Evaluate $\int_0^{\pi/6} 24 \sin^5 \theta \cos \theta d\theta$.
[0.0625]

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Exercise 154

- **Exercise 3.** Integrate with respect to the variable:
 $4\sec^2(3t + 1)$
 $[4\tan(3t + 1)/3 + c]$
- **Exercise 8.** Evaluate the definite integral correct to 4 significant figures: $\int_0^2 x\sqrt{(2x^2 + 1)} dx$
 $[4.333]$

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Further Worked Problems

- **Problem 8.** Determine $\int \frac{2x}{\sqrt{(4x^2 - 1)}} dx$.
 $[\frac{1}{2}\sqrt{(4x^2 - 1)} + c]$
- **Problem 9.** Show that $\int \tan \theta d\theta = \ln(\sec \theta) + c$

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Change of Limits

- When evaluating definite integrals involving substitutions it is sometimes more convenient to **change the limits** of the integral.
- **Problem 10.** Evaluate $\int_1^3 5x\sqrt{(2x^2 + 7)} dx$, taking positive values of square roots only.
 $[245/3]$

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Exercise 155

Integrate with respect to the variable.

- **Exercise 3.** $3\sec^2 3x \tan 3x$
 $[\frac{1}{2}\sec^2 3x + c \text{ or } \frac{1}{2}\tan^2 3x + c]$
- **Exercise 5.** $\frac{\ln \theta}{\theta}$
 $[\frac{\ln^2 \theta}{2} + c]$
- **Exercise 6.** $3\tan 2t$
 $[\frac{3}{2}\ln(\sec 2t) + c]$

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Exercise 155

- **Exercise 10.** Evaluate the definite integral correct to 4 significant figures: $\int_0^1 \frac{3x}{(4x^2 - 1)^5} dx$
[0.09259]

- **Exercise 12.** In the study of a rigid rotor the following integration occurs:

$$Z_r = \int_0^{\infty} (2J + 1) e^{\frac{-J(J+1)h^2}{8\pi^2 IkT}} dJ$$

Determine Z_r for constant temperature T assuming h , I , and k are constants.

$$\left[\frac{8\pi^2 IkT}{h^2} \right]$$