Chapter 37: Standard Integration

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Outline

- The General Solution of Integrals of the Form ax^n
- Standard Integrals
- Definite Integrals

The General Solution of Integrals of the Form ax^n

- The general solution of integrals of the form where a and n are constants is given by: $\int ax^n \mathrm{d}x$

$$\int ax^n \mathrm{d}x = \frac{ax^{n+1}}{n+1} + c$$

This rule is true when n is fractional, zero, or a positive or negative integer, with the exception of $n = -1$.

$$\int \frac{2}{x^2} \mathrm{d}x = \frac{-2}{x} + c$$

$$\int \sqrt{x} \, \mathrm{d}x = \frac{2}{3}\sqrt{x^3} + c$$

Standard Integrals (1/2)

$$\int ax^n \mathrm{d}x = \frac{ax^{n+1}}{n+1} + c \text{ (except when n} = -1)$$

$$\int \cos ax \, \mathrm{d}x = \frac{1}{a}\sin ax + c$$

$$\int \sin ax \, \mathrm{d}x = -\frac{1}{a}\cos ax + c$$

$$\int \sec^2 ax \, \mathrm{d}x = \frac{1}{a}\tan ax + c$$

$$\int \mathrm{cosec}^2 ax \, \mathrm{d}x = -\frac{1}{a}\cot ax + c$$

Standard Integrals (2/2)

$$\int \operatorname{cosec} ax \cot ax \, dx = -\frac{1}{a}\operatorname{cosec} ax + c$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax + c$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \ln x + c$$

Problems

- **Problem 7.** Determine $\int \frac{(1+\theta)^2}{\sqrt{\theta}} d\theta$.

  $[2\sqrt{\theta} + \frac{4}{3}\sqrt{\theta^3} + \frac{2}{5}\sqrt{\theta^5} + c]$

- **Problem 9.** Determine
  (a) $\int 7\sec^2 4t \, dt$ (b) $3\int \operatorname{cosec}^2 2\theta \, d\theta$

  $[(a)\ \frac{7}{4}\tan 4t + c \quad (b) -\frac{3}{2}\cot 2\theta + c]$

Exercise 146

Determine the indefinite integrals.

- **Exercise 5.** (a) $2\int \sqrt{x^3}\, dx$ (b) $\int \frac{1}{4}\sqrt[4]{x^5}\, dx$

  $[(a)\ \frac{4}{5}\sqrt{x^5} + c \quad (b)\ \frac{1}{9}\sqrt[4]{x^9} + c]$

- **Exercise 8.** (a) $\int \frac{3}{4}\sec^2 3x \, dx$ (b) $\int 2\operatorname{cosec}^2 4\theta \, d\theta$

  $[(a)\ \frac{1}{4}\tan 3x + c \quad (b) -\frac{1}{2}\cot 4\theta + c]$

- **Exercise 12.** $\int \frac{(2+3x)^2}{\sqrt{x}} dx$

  $[8\sqrt{x} + 8\sqrt{x^3} + \frac{18}{5}\sqrt{x^5} + c]$

Definite Integrals

- Integrals containing an arbitrary constant c in their results are called indefinite integrals since their precise value cannot be determined without further information.
- Definite integrals are those in which limits are applied.
  - Note that the "c" term always cancels out when limits are applied and it need not be shown with definite integrals.

Problems & Exercise 147

- **Problem 14.** Evaluate $\int_0^{\frac{\pi}{2}} 3\sin 2x \, \mathrm{d}x$.
  [3]

Evaluate the definite integrals (where necessary, correct to 4 significant figures).

- **Exercise 6.** (a) $\int_1^2 \mathrm{cosec}^2 4t \, \mathrm{d}t$

  (b) $\int_{\pi/4}^{\pi/2} (3\sin 2x - 2\cos 3x) \, \mathrm{d}x$

  [(a) 0.2527 (b) 2.638]

Exercise 147

- **Exercise 9.** The entropy change ΔS, for an ideal gas is given by:

  $$\Delta S = \int_{T_1}^{T_2} C_v \frac{\mathrm{d}T}{T} - R\int_{V_1}^{V_2} \frac{\mathrm{d}V}{V}$$

  where T is the thermodynamic temperature, V is the volume and R = 8.314. Determine the entropy change when a gas expands from 1 liter to 3 liters for a temperature rise from 100 K to 400 K given that: $Cv = 45 + 6 \times 10^{-3}T + 8 \times 10^{-6}T^2$

  [55.65]