

# Chapter 32: Differentiation of Hyperbolic Functions

謝仁偉 助理教授

[jenwei@mail.ntust.edu.tw](mailto:jenwei@mail.ntust.edu.tw)

國立台灣科技大學 資訊工程系  
2008 Spring

1

## Outline

- Standard Differential Coefficients of Hyperbolic Functions
- Further Worked Problems on Differentiation of Hyperbolic Functions

2

## Standard Differential Coefficients of Hyperbolic Functions

- If  $y = \sinh ax$ , where 'a' is a constant, then
$$\frac{dy}{dx} = a \cosh ax$$
- If  $y = \cosh ax$ , where 'a' is a constant, then
$$\frac{dy}{dx} = a \sinh ax$$
- Using [the quotient rule of differential](#), the derivatives of  $\tanh x$ ,  $\operatorname{sech} x$ ,  $\operatorname{cosech} x$ , and  $\operatorname{coth} x$  may be determined using the above results.

3

## Problems

- **Problem 1.** Determine the differential coefficient of: (a)  $\operatorname{th} x$  (b)  $\operatorname{sech} x$ .  
[(a)  $\operatorname{sech}^2 x$  (b)  $-\operatorname{sech} x \operatorname{th} x$ ]
- **Problem 2.** Determine  $\frac{dy}{dx}$  given  
(a)  $y = \operatorname{cosech} x$  (b)  $y = \operatorname{coth} x$ .  
[(a)  $-\operatorname{cosech} x \operatorname{coth} x$  (b)  $-\operatorname{cosech}^2 x$ ]

4

## Summary of Differential Coefficients

$f(x)$	$f'(x)$
$\sinh ax$	$a \cosh ax$
$\cosh ax$	$a \sinh ax$
$\tanh ax$	$a \operatorname{sech}^2 ax$
$\operatorname{sech} ax$	$-a \operatorname{sech} ax \tanh ax$
$\operatorname{cosech} ax$	$-a \operatorname{cosech} ax \coth ax$
$\coth ax$	$-a \operatorname{cosech}^2 ax$

5

## Further Worked Problems

- **Problem 4.** Differentiate the following with respect to the variable: (a)  $y = 4 \sin 3x \operatorname{ch} 4x$  (b)  $y = \ln(\operatorname{sh} 3x) - 4\operatorname{ch}^2 3x$ .

$$[(a) 4(4 \sin 3x \operatorname{sh} 4x + 3 \cos 3x \operatorname{ch} 4x)$$

$$(b) 3(\operatorname{coth} 3x - 8 \operatorname{ch} 3x \operatorname{sh} 3x)]$$

- **Problem 5.** Show that the differential coefficient of  $y = \frac{3x^2}{\operatorname{ch} 4x}$  is:  $6x \operatorname{sech} 4x(1 - 2x \operatorname{th} 4x)$

6

## Exercise 135

Differentiate the given functions with respect to the variable:

- **Exercise 3.**  $\frac{3}{4} \ln \left( \operatorname{th} \left( \frac{x}{2} \right) \right)$

$$\left[ \frac{3}{8} \operatorname{sech} \frac{x}{2} \operatorname{cosech} \frac{x}{2} \right]$$

- **Exercise 4.**  $3e^{2x} \operatorname{th} 2x$

$$[6e^{2x}(\operatorname{sech}^2 2x + \operatorname{th} 2x)]$$

7