Chapter 27: Methods of Differentiation

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Outline

- Differentiation of Common Functions
- Differentiation of a Product
- Differentiation of a Quotient
- Function of a Function
- Successive Differentiation

Differentiation of Common Functions

- Summary of Standard Derivatives

$f(x)$	$f'(x)$
ax^n	anx^{n-1}
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$
e^{ax}	ae^{ax}
$\ln ax$	$1/x$

- **Problem 4.** Find the derivatives of
  (a) $y = 3\sqrt{x}$ (b) $y = 5/\sqrt[3]{x^4}$
  [(a) $3/2\sqrt{x}$ (b) $-20/3\sqrt[3]{x^7}$]

Problems

- **Problem 6.** Find the differential coefficients of (a) $y = 3\sin 4x$ (b) $f(t) = 2\cos 3t$ with respect to the variable.
  [(a) $12\cos 4x$ (b) $-6\sin 3t$]
- **Problem 7.** Determine the derivatives of (a) $y = 3e^{5x}$ (b) $f(x) = 2/e^{3x}$ (c) $y = 6\ln 2x$.
  [(a) $15e^{5x}$ (b) $-6/e^{3x}$ (c) $6/x$]
- **Problem 9.** Determine the co-ordinates of the point on the graph $y = 3x^2 - 7x + 2$ where the gradient is –1.
  [(1, – 2)]

Exercise 117

Find the differential coefficients of the given functions with respect to the variable.

- **Exercise 4.** (a) $-3/\sqrt[3]{x}$ (b) $(x-1)^2$ (c) $2\sin 3x$

  [(a) $1/\sqrt[3]{x^4}$ (b) $2(x-1)$ (c) $6\cos 3x$]

- **Exercise 6.** (a) $4\ln 9x$ (b) $\dfrac{e^x - e^{-x}}{2}$ (c) $\dfrac{1-\sqrt{x}}{x}$

  [(a) $\dfrac{4}{x}$ (b) $\dfrac{e^x + e^{-x}}{2}$ (c) $\dfrac{-1}{x^2} + \dfrac{1}{2\sqrt{x^3}}$]

Differentiation of a Product

- Product rule: When $y = uv$, and u and v are both functions of x, then

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

- **Problem 12.** Differentiate $y = x^3\cos 3x \ln x$.

  [$x^2\{\cos 3x + 3 \ln x (\cos 3x - x \sin 3x)\}$]

Exercise 118

- **Exercise 5.** Differentiate the given products with respect to the variable: $e^t \ln t \cos t$

  [$e^t\{(1/t + \ln t)\cos t - \ln t \sin t\}$]

- **Exercise 7.** Evaluate dz/dt, correct to 4 significant figures, when $t = 0.5$, given that $z = 2e^{3t}\sin 2t$.

  [32.31]

Differentiation of a Quotient

- Quotient rule: When $y = u/v$, and u and v are both functions of x, then

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

- **Problem 14.** Find the differential coefficient of $y = 4\sin 5x / 5x^4$.

  [$4(5x\cos 5x - 4\sin 5x)/5x^5$]

- **Problem 15.** Determine the differential coefficient of $y = \tan ax$.

  [$a\sec^2 ax$]

Problems

- **Problem 16.** Find the derivative of $y = \sec ax$.
  [a sec ax tan ax]

- **Problem 17.** Differentiate $y = te^{2t} / 2\cos t$.
  [e^{2t}(2t cos t + cos t + t sin t) / 2 $\cos^2$ t]

- **Problem 18.** Determine the gradient of the curve $y = 5x / (2x^2 + 4)$ at the point $(\sqrt{3}, \sqrt{3}/2)$.
  [-1/10]

Exercise 119

Differentiate quotients with respect to the variable.

- **Exercise 3.** $\dfrac{3\sqrt{x^3}}{2\sin 2x}$
  $\left[\dfrac{3\sqrt{x}(3\sin 2x - 4x\cos 2x}{4\sin^2 2x}\right]$
- **Exercise 5.** $\dfrac{2xe^{4x}}{\sin x}$
  $\left[\dfrac{2e^{4x}}{\sin^2 x}\{(1+4x)\sin x - x\cos x\}\right]$

Function of a Function

- Function of a function rule or chain rule: If y is a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

- **Problem 19.** Differentiate y = 3 cos(5x^2 + 2).
  [-30x sin(5x^2 + 2)]
- **Problem 20.** Find the derivative of y = (4t^3 – 3t)^6.
  [18(4t^2 - 1)(4t^3 - 3t)^5]
- **Problem 22.** Differentiate y = 3 tan^{4}3x.
  [36 tan^{3}3x sec^{2}3x]

Exercise 120

Find the differential coefficients with respect to the variable.

- **Exercise 3.** 2 cos^5 x
  [-10 cos^4 x sin x]
- **Exercise 6.** 2 cot (5t^2 + 3)
  [-20t cosec^2(5t^2 + 3)]
- **Exercise 8.** 2e$^{\tan x}$
  [2 sec^2 x e$^{\tan x}$]

Successive Differentiation

- **Problem 24.** If $f(x) = 2x^5 - 4x^3 + 3x - 5$, find $f''(x)$. ($f''(x)$ pronounced f double-dash x)
  [$4x(10x^2 - 6)$]

- **Problem 27.** Evaluate d^2y/dx^2 when $x = 0$ given $y = 4 \sec 2x$. (d^2y/dx^2 pronounced dee two y by dee x squared)
  [$d^2y/dx^2 = 16 \sec^3 2x + 16 \sec 2x \tan^2 2x$, 16]

Exercise 121

In Exercise 3 and 4, find the second differential coefficient with respect to the variable.

- **Exercise 3.** (a) $3 \sin 2t + \cos t$ (b) $2 \ln 4x$
  [(a) $-(12 \sin 2t + \cos t)$ (b) $-2/x^2$]
- **Exercise 4.** (a) $2 \cos^2 x$ (b) $(2x - 3)^4$
  [(a) $4(\sin^2 x - \cos^2 x)$ (b) $48(2x - 3)^2$]
- **Exercise 7.** Show that, if P and Q are constants and $y = P \cos(\ln t) + Q \sin(\ln t)$, then

$$t^2 \frac{d^2y}{dt^2} + t \frac{dy}{dt} + y = 0$$