# Chapter 11: Boolean Algebra and Logic Circuits

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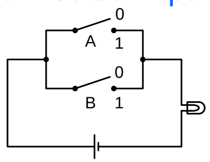
# Outline

- Boolean Algebra and Switching Circuits
- Laws and Rules of Boolean Algebra
- De Morgan's Laws
- Karnaugh Maps
- Logic Circuits
- Universal Logic Gates



# Boolean Algebra and Switching Circuits (1/2)

- A two-state device is one whose basic elements can only have one of two conditions.
- In Boolean algebra, if $A$ represents one state, then $\overline{A}$, called 'not-$A$', represents the second state.
- The or-function: $A + B$
  - Defined as 'A, or B, or both A and B'
  - The equivalent electrical circuit is two switches connected in parallel.



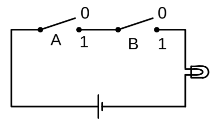

| A | B | Z = A + B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Truth Table



# Boolean Algebra and Switching Circuits (2/2)

- The and-function: $A \bullet B$
  - Defined as 'both A and B'
  - The equivalent electrical circuit is two switches connected in series.



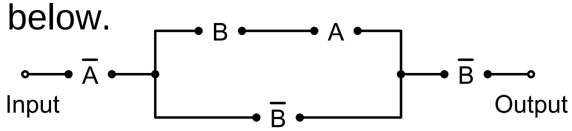

| A | B | Z = A • B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

- The not-function: $\overline{A}$
  - Defined as 'the opposite to A'

| A | $Z = \overline{A}$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

# Problems

- **Problem 1.** Derive the Boolean expression and construct a truth table for the switching circuit shown below.




$$[Z = \bar{A}\cdot(B\cdot A + \bar{B})\cdot\bar{B}]$$

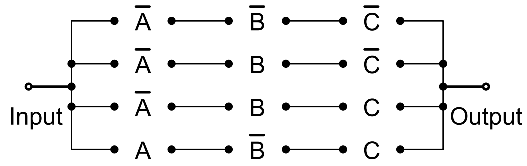
| A | B | B•A | $\bar{B}$ | B•A+$\bar{B}$ | $\bar{A}$ | Z=$\bar{A}$•(B•A+$\bar{B}$)•$\bar{B}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 |



# Problems

- **Problem 4.** Derive the Boolean expression and construct the switching circuit for the truth table given below.

| | A | B | C | Z |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 0 | 1 | 0 | 1 |
| 4 | 0 | 1 | 1 | 1 |
| 5 | 1 | 0 | 0 | 0 |
| 6 | 1 | 0 | 1 | 1 |
| 7 | 1 | 1 | 0 | 0 |
| 8 | 1 | 1 | 1 | 0 |

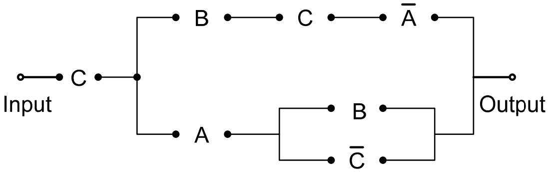



$$[Z = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + \bar{A}\cdot B\cdot C + A\cdot\bar{B}\cdot C]$$



# Exercise 46

- **Exercise 4.** Determine the Boolean expressions and construct truth table for the switching circuit.



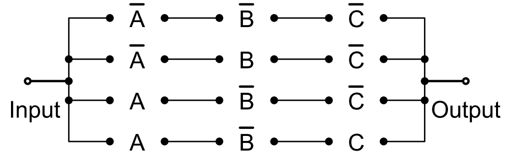

| | A | B | C | Z |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 0 | 1 | 0 | 0 |
| 4 | 0 | 1 | 1 | 1 |
| 5 | 1 | 0 | 0 | 0 |
| 6 | 1 | 0 | 1 | 0 |
| 7 | 1 | 1 | 0 | 0 |
| 8 | 1 | 1 | 1 | 1 |

$$[Z = C\cdot(B\cdot C\cdot\bar{A} + A\cdot(B + \bar{C}))]$$



# Exercise 46

- **Exercise 10.** Derive the Boolean expression and construct the switching circuit for the truth table stated.




| | A | B | C | Z |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 0 | 1 | 0 | 1 |
| 4 | 0 | 1 | 1 | 0 |
| 5 | 1 | 0 | 0 | 1 |
| 6 | 1 | 0 | 1 | 1 |
| 7 | 1 | 1 | 0 | 0 |
| 8 | 1 | 1 | 1 | 0 |

$$[Z = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot\bar{C} + A\cdot\bar{B}\cdot C]$$

## Simplifying Boolean Expressions

- If the Boolean expression can be simplified, then the number of switches or logic elements can be reduced resulting in a saving in cost.
- Three principal ways of simplifying Boolean expression are:
  - By using the laws and rules of Boolean algebra
  - By applying de Morgan's laws
  - By using Karnaugh maps



## Laws and Rules of Boolean Algebra (1/2)

- Commutative Laws $A + B = B + A$
  $A \cdot B = B \cdot A$
- Associative Laws $(A + B) + C = A + (B + C)$
  $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
- Distributive Laws $A \cdot (B + C) = A \cdot B + A \cdot C$
  $A + (B \cdot C) = (A + B) \cdot (A + C)$
- Sum Rules $A + 0 = A$
  $A + 1 = 1$
  $A + A = A$
  $A + \overline{A} = 1$



## Laws and Rules of Boolean Algebra (2/2)

- Product Rules $A \cdot 0 = 0$
  $A \cdot 1 = A$
  $A \cdot A = A$
  $A \cdot \overline{A} = 0$
- Absorption Rules $A + A \cdot B = A$
  $A \cdot (A + B) = A$
  $A + \overline{A} \cdot B = A + B$



## Problems

- **Problem 5.** Simplify the Boolean expression:
  $\overline{P} \cdot \overline{Q} + \overline{P} \cdot Q + P \cdot \overline{Q}$
  $[\overline{P} + \overline{Q}]$
- **Problem 6.** Simplify $(P + \overline{P} \cdot Q) \cdot (Q + \overline{Q} \cdot P)$
  $[P + Q]$
- **Problem 7.** Simplify
  $F \cdot G \cdot \overline{H} + F \cdot G \cdot H + \overline{F} \cdot G \cdot H$
  $[G \cdot (F + H)]$

## Problems

- **Problem 8.** Simplify
  $\bar{F}\bullet\bar{G}\bullet H+\bar{F}\bullet G\bullet H+F\bullet\bar{G}\bullet H+F\bullet G\bullet H$
  [$H$]

- **Problem 9.** Simplify
  $A\bullet\bar{C}+\bar{A}\bullet(B+C)+A\bullet B\bullet(C+\bar{B})$
  [$A\cdot\bar{C}+B+\bar{A}\cdot C$]

- **Problem 10.** Simplify
  $P\bullet\bar{Q}\bullet R+P\bullet Q\bullet(\bar{P}+R)+Q\bullet R\bullet(\bar{Q}+P)$
  [$P\cdot R$]



## Exercise 47

Use the laws and rules of Boolean algebra to simplify the following expressions:

- **Exercise 6.** $\bar{F}\bullet\bar{G}\bullet H+\bar{F}\bullet G\bullet H+F\bullet\bar{G}\bullet H$
  [$H\cdot(\bar{F}+\bar{G})$]

- **Exercise 12.**
  $\bar{R}\bullet(\bar{P}\bullet\bar{Q}+P\bullet Q+P\bullet\bar{Q})+P\bullet(Q\bullet R+\bar{Q}\bullet R)$
  [$P+\bar{Q}\cdot\bar{R}$]



## De Morgan's Laws & Problems

- De Morgan's laws may be used to simplify not-functions having two or more elements.
- The laws state that:

$$\overline{A+B}=\bar{A}\cdot\bar{B} \quad \text{and} \quad \overline{A\cdot B}=\bar{A}+\bar{B}$$

- **Problem 11.** Verify that $\overline{A+B}=\bar{A}\cdot\bar{B}$
- **Problem 12.** Simplify the Boolean expression $\overline{(\bar{A}\cdot B)}+\overline{(\bar{A}+B)}$ by using de Morgan's laws and the rules of Boolean algebra.
  [$A+\bar{B}$]



## Problems & Exercise 48

- **Problem 13.** Simplify the Boolean expression $(\overline{A\cdot\bar{B}+C})\cdot(\bar{A}+\overline{B\cdot\bar{C}})$ by using de Morgan's laws and the rules of Boolean algebra.
  [$\bar{A}\cdot\bar{C}$]

Use de Morgan's law and the rules of Boolean algebra to simplify the following expressions.

- **Exercise 2.** $(\overline{A+\overline{B\cdot C}})+(\overline{A\cdot B}+C)$
  [$\bar{A}+\bar{B}+C$]
- **Exercise 3.** $(\overline{\bar{A}\cdot B+B\cdot\bar{C}})\cdot\overline{A\cdot\bar{B}}$
  [$\bar{A}\cdot\bar{B}+A\cdot B\cdot C$]

# Two-Variable Karnaugh Maps

| Inputs | | Output | Boolean |
|---|---|---|---|
| A | B | Z | expression |
| 0 | 0 | 0 | $\overline{A}\cdot\overline{B}$ |
| 0 | 1 | 0 | $\overline{A}\cdot B$ |
| 1 | 0 | 1 | $A\cdot\overline{B}$ |
| 1 | 1 | 0 | $A\cdot B$ |

A truth table for a two-variable expression

| B \ A | 0 ($\overline{A}$) | 1 (A) |
|---|---|---|
| 0 ($\overline{B}$) | $\overline{A}\cdot\overline{B}$ | $A\cdot\overline{B}$ |
| 1 (B) | $\overline{A}\cdot B$ | $A\cdot B$ |

Each of the four possible Boolean expression associated with a two-variable function

| B \ A | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0 | 0 |

Two-variable Karnaugh map



# Three-Variable Karnaugh Maps

A truth table for the three-variable expression:

$$Z=\overline{A}\cdot\overline{B}\cdot C+\overline{A}\cdot B\cdot C+A\cdot B\cdot\overline{C}$$

| Inputs | | | Output | Boolean |
|---|---|---|---|---|
| A | B | C | Z | expression |
| 0 | 0 | 0 | 0 | $\overline{A}\cdot\overline{B}\cdot\overline{C}$ |
| 0 | 0 | 1 | 1 | $\overline{A}\cdot\overline{B}\cdot C$ |
| 0 | 1 | 0 | 0 | $\overline{A}\cdot B\cdot\overline{C}$ |
| 0 | 1 | 1 | 1 | $\overline{A}\cdot B\cdot C$ |
| 1 | 0 | 0 | 0 | $A\cdot\overline{B}\cdot\overline{C}$ |
| 1 | 0 | 1 | 0 | $A\cdot\overline{B}\cdot C$ |
| 1 | 1 | 0 | 1 | $A\cdot B\cdot\overline{C}$ |
| 1 | 1 | 1 | 0 | $A\cdot B\cdot C$ |

| C \ A•B | 00 ($\overline{A}\cdot\overline{B}$) | 01 ($\overline{A}\cdot B$) | 11 ($A\cdot B$) | 10 ($A\cdot\overline{B}$) |
|---|---|---|---|---|
| 0 ($\overline{C}$) | $\overline{A}\cdot\overline{B}\cdot\overline{C}$ | $\overline{A}\cdot B\cdot\overline{C}$ | $A\cdot B\cdot\overline{C}$ | $A\cdot\overline{B}\cdot\overline{C}$ |
| 1 (C) | $\overline{A}\cdot\overline{B}\cdot C$ | $\overline{A}\cdot B\cdot C$ | $A\cdot B\cdot C$ | $A\cdot\overline{B}\cdot C$ |

| C \ A•B | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |



# Four-Variable Karnaugh Maps

| Inputs | | | | Output | Boolean |
|---|---|---|---|---|---|
| A | B | C | D | Z | expression |
| 0 | 0 | 0 | 0 | 0 | $\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}$ |
| 0 | 0 | 0 | 1 | 0 | $\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot D$ |
| 0 | 0 | 1 | 0 | 1 | $\overline{A}\cdot\overline{B}\cdot C\cdot\overline{D}$ |
| 0 | 0 | 1 | 1 | 0 | $\overline{A}\cdot\overline{B}\cdot C\cdot D$ |
| 0 | 1 | 0 | 0 | 0 | $\overline{A}\cdot B\cdot\overline{C}\cdot\overline{D}$ |
| 0 | 1 | 0 | 1 | 0 | $\overline{A}\cdot B\cdot\overline{C}\cdot D$ |
| 0 | 1 | 1 | 0 | 1 | $\overline{A}\cdot B\cdot C\cdot\overline{D}$ |
| 0 | 1 | 1 | 1 | 0 | $\overline{A}\cdot B\cdot C\cdot D$ |
| 1 | 0 | 0 | 0 | 0 | $A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}$ |
| 1 | 0 | 0 | 1 | 0 | $A\cdot\overline{B}\cdot\overline{C}\cdot D$ |
| 1 | 0 | 1 | 0 | 1 | $A\cdot\overline{B}\cdot C\cdot\overline{D}$ |
| 1 | 0 | 1 | 1 | 0 | $A\cdot\overline{B}\cdot C\cdot D$ |
| 1 | 1 | 0 | 0 | 0 | $A\cdot B\cdot\overline{C}\cdot\overline{D}$ |
| 1 | 1 | 0 | 1 | 0 | $A\cdot B\cdot\overline{C}\cdot D$ |
| 1 | 1 | 1 | 0 | 1 | $A\cdot B\cdot C\cdot\overline{D}$ |
| 1 | 1 | 1 | 1 | 0 | $A\cdot B\cdot C\cdot D$ |

$$Z=\overline{A}\cdot\overline{B}\cdot C\cdot\overline{D}+\overline{A}\cdot B\cdot C\cdot\overline{D}$$
$$+A\cdot\overline{B}\cdot C\cdot\overline{D}+A\cdot B\cdot C\cdot\overline{D}$$

| C•D \ A•B | 00 ($\overline{A}\cdot\overline{B}$) | 01 ($\overline{A}\cdot B$) | 11 ($A\cdot B$) | 10 ($A\cdot\overline{B}$) |
|---|---|---|---|---|
| 00 ($\overline{C}\cdot\overline{D}$) | $\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}$ | $\overline{A}\cdot B\cdot\overline{C}\cdot\overline{D}$ | $A\cdot B\cdot\overline{C}\cdot\overline{D}$ | $A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}$ |
| 01 ($\overline{C}\cdot D$) | $\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot D$ | $\overline{A}\cdot B\cdot\overline{C}\cdot D$ | $A\cdot B\cdot\overline{C}\cdot D$ | $A\cdot\overline{B}\cdot\overline{C}\cdot D$ |
| 11 ($C\cdot D$) | $\overline{A}\cdot\overline{B}\cdot C\cdot D$ | $\overline{A}\cdot B\cdot C\cdot D$ | $A\cdot B\cdot C\cdot D$ | $A\cdot\overline{B}\cdot C\cdot D$ |
| 10 ($C\cdot\overline{D}$) | $\overline{A}\cdot\overline{B}\cdot C\cdot\overline{D}$ | $\overline{A}\cdot B\cdot C\cdot\overline{D}$ | $A\cdot B\cdot C\cdot\overline{D}$ | $A\cdot\overline{B}\cdot C\cdot\overline{D}$ |

| C•D \ A•B | 0•0 | 0•1 | 1•1 | 1•0 |
|---|---|---|---|---|
| 0•0 | 0 | 0 | 0 | 0 |
| 0•1 | 0 | 0 | 0 | 0 |
| 1•1 | 0 | 0 | 0 | 0 |
| 1•0 | 1 | 1 | 1 | 1 |



# Simplifying a Boolean Expression Using a Karnaugh Map (1/2)

a) Draw a 4, 8, or 16-cell matrix, depending on whether there are 2, 3, or 4 variables.

b) Mark in the Boolean expression by putting 1's in the appropriate cells.

c) From couples of 8, 4, or 2 cells having common edges, forming the largest groups of cells possible.

- Note that a cell containing a 1 may be used more than once when forming a couple.
- Also note that each cell containing a 1 must be used at least once.

# Simplifying a Boolean Expression Using a Karnaugh Map (2/2)

d) The Boolean expression for the couple is given by the variables which are common to all cells in the couple.



# Problems

- **Problem 14.** Use the Karnaugh map techniques to simplify the expression $\overline{P}\cdot\overline{Q}+\overline{P}\cdot Q$
  $[\overline{P}]$
- **Problem 15.** Simplify the expression
  $\overline{X}\cdot Y\cdot\overline{Z}+\overline{X}\cdot\overline{Y}\cdot Z+X\cdot Y\cdot\overline{Z}+X\cdot\overline{Y}\cdot Z$
  by using Karnaugh map techniques.
  $[Y\cdot\overline{Z}+\overline{Y}\cdot Z]$
- **Problem 17.** Simplify $\overline{(P+\overline{Q}\cdot R)}+\overline{(P\cdot Q+\overline{R})}$ using a Karnaugh map technique.
  $[\overline{P}+\overline{Q}\cdot R]$



# Problems

- **Problem 19.** Simplify the expression $\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}+$
  $A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}+\overline{A}\cdot\overline{B}\cdot C\cdot\overline{D}+A\cdot\overline{B}\cdot C\cdot D+A\cdot B\cdot C\cdot D$
  by using Karnaugh map techniques.
  $[\overline{B}\cdot\overline{D}+A\cdot B\cdot C\cdot D]$



# Exercise 49

Use Karnaugh map techniques to simplify the expression given.

- **Exercise 3.** $(\overline{P}\cdot\overline{Q})\cdot(\overline{\overline{P}\cdot Q})$
  $[\overline{P}\cdot\overline{Q}]$
- **Exercise 4.** $A\cdot\overline{C}+\overline{A}\cdot(B+C)+A\cdot B\cdot(C+\overline{B})$
  $[A\cdot\overline{C}+B+\overline{A}\cdot C]$
- **Exercise 8.** $\overline{A}\cdot\overline{B}\cdot C\cdot D+\overline{A}\cdot\overline{B}\cdot C\cdot\overline{D}+A\cdot\overline{B}\cdot C\cdot\overline{D}$
  $[\overline{B}\cdot C\cdot(\overline{A}+\overline{D})]$

# Exercise 49

Use Karnaugh map techniques to simplify the expression given.

- **Exercise 10.** $\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot D + A\cdot B\cdot\overline{C}\cdot\overline{D} + A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D} + A\cdot B\cdot C\cdot\overline{D} + A\cdot\overline{B}\cdot C\cdot D$

  $[A\cdot\overline{D} + \overline{A}\cdot\overline{B}\cdot\overline{C}\cdot D]$

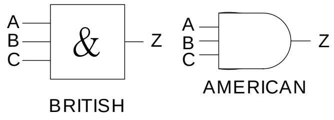
- **Exercise 11.** $A\cdot B\cdot\overline{C}\cdot\overline{D} + \overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\overline{D} + \overline{A}\cdot B\cdot C\cdot D + \overline{A}\cdot\overline{B}\cdot C\cdot D + A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D} + \overline{A}\cdot\overline{B}\cdot C\cdot\overline{D} + \overline{A}\cdot B\cdot C\cdot\overline{D}$

  $[\overline{A}\cdot C + A\cdot\overline{C}\cdot\overline{D} + \overline{B}\cdot\overline{C}\cdot\overline{D}]$



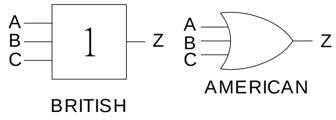
# Logic Circuits (1/3)

- The and-gate:




| INPUTS | | | OUTPUT |
|---|---|---|---|
| A | B | C | Z = A • B • C |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

- The or-gate:

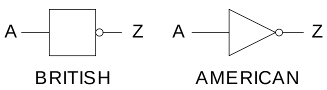



| INPUTS | | | OUTPUT |
|---|---|---|---|
| A | B | C | Z = A + B + C |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |



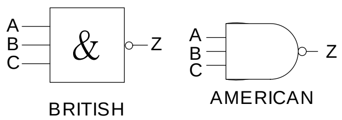
# Logic Circuits (2/3)

- The invert-gate or not-gate:




| INPUTS | OUTPUT |
|---|---|
| A | Z = $\overline{A}$ |
| 0 | 1 |
| 1 | 0 |

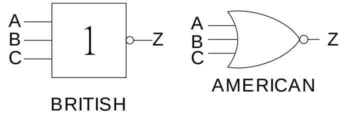
- The nand-gate:




| INPUTS | | | A • B • C | OUTPUT |
|---|---|---|---|---|
| A | B | C | | Z = $\overline{A \bullet B \bullet C}$ |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

# Logic Circuits (3/3)

- The nor-gate:




| INPUTS | | | A + B + C | OUTPUT |
|---|---|---|---|---|
| A | B | C | | Z = $\overline{A + B + C}$ |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 |

# Combinational Logic Networks (1/2)

- **Problem 22.** Device a logic circuit to meet the requirements of the output given below using as few gates as possible.
  $[Z = A \cdot (B + C)]$

| Inputs | | | Output |
|---|---|---|---|
| A | B | C | Z |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |



# Combinational Logic Networks (2/2)

- **Problem 23.** Simplify the expression:
  $$Z = \overline{P}\cdot\overline{Q}\cdot\overline{R}\cdot\overline{S} + \overline{P}\cdot\overline{Q}\cdot\overline{R}\cdot S + \overline{P}\cdot Q\cdot\overline{R}\cdot\overline{S} + \overline{P}\cdot Q\cdot\overline{R}\cdot S$$
  $$+ P\cdot\overline{Q}\cdot\overline{R}\cdot\overline{S}$$
  and devise a logic circuit to give this output.
  $[Z = \overline{R}\cdot(\overline{P} + \overline{Q}\cdot\overline{S})]$



# Exercise 50

- **Exercise 2.** Devise a logic system to meet the requirement of the Boolean expression:
  $Z = A\cdot\overline{B} + B\cdot\overline{C}$
- **Exercise 5.** Simplify the expression given in the truth table and devise a logic circuit to meet the requirements stated.
  $[Z = A\cdot B + C]$

| A | B | C | Z |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |



# Exercise 50

Simplify the Boolean expressions given and devise a logic circuit to give the requirement of the simplified expression.

- **Exercise 9.** $\overline{P}\cdot\overline{Q}\cdot\overline{R} + P\cdot Q\cdot\overline{R} + P\cdot\overline{Q}\cdot\overline{R}$
  $[\overline{R}\cdot(P + \overline{Q})]$
- **Exercise 11.** $\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\overline{D} + A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D} + \overline{A}\cdot\overline{B}\cdot C\cdot\overline{D}$
  $+ \overline{A}\cdot B\cdot C\cdot\overline{D} + A\cdot\overline{B}\cdot C\cdot\overline{D}$
  $[\overline{D}\cdot(\overline{A}\cdot C + \overline{B})]$

## Exercise 50

- **Exercise 12.** $\overline{(\overline{P}\cdot Q\cdot R)}\cdot\overline{(P+Q\cdot R)}$

  $[\overline{P}\cdot(\overline{Q}+\overline{R})]$



## Universal Logic Gates & Problem

- The function of any of the five logic gates in common use can be obtained by using either nand-gates or nor-gates.
- When used in this manner, the gate selected is called a universal gate.
- **Problem 24.** Show how invert, and, or, and nor-functions can be produced using nand-gates only.
- **Problem 25.** Show how invert, or, and, and nand-functions can be produced by using nor-gates only.



## Problems

- **Problem 26.** Design a logic circuit, using nand-gates having not more than three inputs, to meet the requirements of the Boolean expression

$$Z=\overline{A}+\overline{B}+C+\overline{D}$$

  [hint: It is usual in logic circuit design to start the design at the output.]
- **Problem 27.** Use nor-gates only to design a logic circuit to meet the requirements of the expression: $Z=\overline{D}\cdot(\overline{A}+B+\overline{C})$



## Problems

- **Problem 28.** An alarm indicator in a grinding mill complex should be activated if (a) the power supply to all mills is off and (b) the hopper feeding the mills is less than 10% full, and (c) if one of the three grinding mills is not in action. Device a logic system to meet these requirements.

  [A: the power supply on to all the mills
  B: the hopper feeding the mills being more than 10% full
  C, D, and E: the three mills being in action respectively]

## Exercise 51

- **Exercise 1.** Use nand-gates only to devise the logic system: $Z = A + B \bullet C$

In Problem 4 to 6, use nor-gates only to devise the logic systems stated.

- **Exercise 4.** $Z = (\overline{A} + B) \cdot (\overline{C} + D)$
- **Exercise 5.** $Z = A \cdot \overline{B} + B \cdot \overline{C} + C \cdot \overline{D}$
- **Exercise 6.** $Z = \overline{P} \cdot Q + P \cdot (Q + R)$



## Exercise 51

- **Exercise 9.** A water tank feeds three separate processes. When any two of the processes are in operation at the same time, a signal is required to start a pump to maintain the head of water in the tank. Devise a logic circuit using nor-gates only to give the required signal.
  [$Z = A \cdot (B + C) + B \cdot C$
  or $Z = A \cdot B + A \cdot C + B \cdot C$]



# Homework Assignment 3

Deadline: 21 April 2008
(Firm Real-Time)



# Midterm Announcement

21 April 2008