

Chapter 8: Maclaurin's Series

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1

Outline

- Introduction
- Conditions of Maclaurin's Series
- Worked Problems on Maclaurin's Series
- Numerical Integration Using Maclaurin's Series
- Limiting Values

2

Introduction

- Using **Maclaurin's series**, mixed functions containing, say, algebraic, trigonometric and exponential functions, may be expressed solely as algebraic functions, and differentiation and integration can often be more readily performed.
- **Maclaurin's theorem** or **Maclaurin's series**:

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots$$

3

Conditions of Maclaurin's Series

At $x = 0$ the following 3 conditions are met:

- $f(0) \neq \infty$
- $f'(0), f''(0), f'''(0), \dots \neq \infty$
- The resultant Maclaurin's series must be convergent.

4

Worked Problems on Maclaurin's Series

- **Problem 4.** Expand $\ln(1 + x)$ to five terms.
[$\ln(1 + x) = x - x^2/2 + x^3/3 - x^4/4 + x^5/5 - \dots$]
- **Problem 5.** Expand $\ln(1 - x)$ to five terms.
[$\ln(1 - x) = -x - x^2/2 - x^3/3 - x^4/4 - x^5/5 - \dots$]
- **Problem 6.** Determine the power series for $\ln((1 + x)/(1 - x))$.
[$\ln((1 + x)/(1 - x)) = 2(x + x^3/3 + x^5/5 + \dots)$]

5

Worked Problems on Maclaurin's Series

- **Problem 1.** Determine the first four terms of the power series for $\cos x$.
[$\cos x = 1 - x^2/2! + x^4/4! - x^6/6! + \dots$]
- **Problem 10.** Produce a power series for $\cos^2 2x$ as far as the term in x^6 .
[$\cos^2 2x = 1 - 4x^2 + 16x^4/3 - 128x^6/45 + \dots$]

6

Exercise 36

- **Exercise 1.** Determine the first four terms of the power series for $\sin 2x$ using Maclaurin's series.
[$\sin 2x = 2x - 4x^3/3 + 4x^5/15 - 8x^7/315 + \dots$]
- **Exercise 3.** Use Maclaurin's theorem to determine the first three terms of power series for $\ln(1 + e^x)$.
[$\ln 2 + x/2 + x^2/8$]
- **Exercise 6.** Develop, as far as the term in x^4 , the power series for $\sec 2x$.
[$1 + 2x^2 + 10x^4/3$]

7

Exercise 36

- **Exercise 7.** Expand $e^{2x} \cos 3x$ as far as the term in x^2 using Maclaurin's series.
[$1 + 2x - 5x^2/2$]
- **Exercise 8.** Determine the first three terms of the series for $\sin^2 x$ by applying Maclaurin's theorem.
[$x^2 - x^4/3 + 2x^6/45 \dots$]

8

Numerical Integration Using Maclaurin's Series

- A method of finding the approximate value of a definite integral is to **express the function as a power series** using Maclaurin's series, and then **integrating each algebraic term in turn**.
- **Problem 11.** Evaluate $\int_{0.1}^{0.4} 2e^{\sin\theta} d\theta$, correct to 3 significant figures.
[0.771]
- **Problem 12.** Evaluate $\int_0^1 \frac{\sin\theta}{\theta} d\theta$ using Maclaurin's series, correct to 3 significant figures.
[0.946]

9

Exercise 37

- **Exercise 2.** Use Maclaurin's theorem to expand $\cos 2x$ and hence evaluate, correct to 2 decimal places, $\int_0^1 \frac{\cos 2x}{x^{1/3}} dx$.
[0.88]
- **Exercise 4.** Use Maclaurin's theorem to expand $\sqrt{x} \ln(x+1)$ as a power series. Hence evaluate, correct to 3 decimal places, $\int_0^{0.5} \sqrt{x} \ln(x+1) dx$.
[0.061]

10

Limiting Values (1/2)

- It is sometimes necessary to find limits of the form $\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\}$, where $f(a) = 0$ and $g(a) = 0$.
- For certain limits, a knowledge of series can sometimes help.

$$\begin{aligned} \lim_{x \rightarrow 0} \left\{ \frac{\tan x - x}{x^3} \right\} &\equiv \lim_{x \rightarrow 0} \left\{ \frac{x + x^3/3 + \dots - x}{x^3} \right\} \\ &= \lim_{x \rightarrow 0} \left\{ \frac{x^3/3 + \dots}{x^3} \right\} = \lim_{x \rightarrow 0} \left\{ \frac{1}{3} \right\} = \frac{1}{3} \end{aligned}$$

11

Limiting Values (2/2)

- However, a knowledge of series does not help with examples such as $\lim_{x \rightarrow 1} \left\{ \frac{x^2 + 3x - 4}{x^2 - 7x + 6} \right\}$.
- **L'Hopital's rule** will enable us to determine such limits when the differential coefficients of the numerator and denominator can be found.

12

L'Hopital's Rule

- L'Hopital's rule states:

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ \frac{f'(x)}{g'(x)} \right\} \text{ provided } g'(a) \neq 0.$$

- It can happen that $\lim_{x \rightarrow a} \left\{ \frac{f'(x)}{g'(x)} \right\}$ is still $\frac{0}{0}$; if so, the numerator and denominator are differentiated again (and again) until a non-zero value is obtained for the denominator.

13

Problems & Exercise 38

- Problem 14.** Determine $\lim_{x \rightarrow 1} \left\{ \frac{x^2 + 3x - 4}{x^2 - 7x + 6} \right\}$
[-1]
- Problem 15.** Determine $\lim_{x \rightarrow 0} \left\{ \frac{\sin x - x}{x^2} \right\}$
[0]
- Exercise 5.** Determine $\lim_{\theta \rightarrow 0} \left\{ \frac{\sin \theta - \theta \cos \theta}{\theta^3} \right\}$
[1/3]
- Exercise 7.** Determine $\lim_{x \rightarrow 0} \left\{ \frac{\sinh x - \sin x}{x^3} \right\}$
[1/3]

14

Exercise 38

- Exercise 8.** Determine $\lim_{\theta \rightarrow \frac{\pi}{2}} \left\{ \frac{\sin \theta - 1}{\ln \sin \theta} \right\}$
[1]
- Exercise 9.** Determine $\lim_{t \rightarrow 0} \left\{ \frac{\sec t - 1}{t \sin t} \right\}$
[1/2]

15

Homework Assignment 2

Deadline: **7 April 2008**
(Firm Real-Time)

16