

Chapter 6: Arithmetic and Geometric Progressions

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2008 Spring

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Outline

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- Worked Problems on Arithmetic Progressions
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Arithmetic Progressions

- When a sequence has a **constant difference** between successive terms it is called an **arithmetic progression** (often abbreviated to **AP**).
- If the first term of an AP is ' a ' and the common difference is ' d ' then
 - The n 'th term is: $a + (n - 1)d$
 - The sum of n terms is: $S_n = n[2a + (n - 1)d]/2$

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Worked Problems on Arithmetic Progressions and Exercise 28

- **Problem 2.** The 6th term of an AP is 17 and the 13th term is 38. Determine the 19th term.
[56]
- **Exercise 7.** Determine the sum of the series 6.5, 8.0, 9.5, 11.0, ..., 32
[346.5]

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Further Worked Problems on Arithmetic Progressions

- **Problem 7.** Three numbers are in arithmetic progression. Their sum is 15 and their product is 80. determine the three numbers.
[2, 5, and 8]
- **Problem 9.** The first, twelfth and last term of an arithmetic progression are 4, 31.5, and 376.5, respectively. Determine (a) the number of terms in the series, (b) the sum of all the terms and (c) the '80'th term.
[(a) 150 (b) 28537.5 (c) 201.5]

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Exercise 29

- **Exercise 3.** Find the sum of all the numbers between 5 and 250 which are exactly divisible by 4.
[7808]
- **Exercise 5.** Insert four terms between 5 and 22.5 to form an arithmetic progression.
[8.5, 12, 15.5, 19]

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Geometric Progressions

- When a sequence has a **constant ratio** between successive terms it is called a **geometric progression** (often abbreviated to **GP**).
- If the first term of a GP is ' a ' and the common ratio is ' r ' then
 - The n 'th term is: ar^{n-1}
 - The sum of n terms is:
 - $S_n = a(1 - r^n)/(1 - r)$ for $r < 1$
 - $S_n = a(r^n - 1)/(r - 1)$ for $r > 1$
 - $S_\infty = a/(1 - r)$ for $-1 < r < 1$ and $n \rightarrow \infty$
The quantity $a/(1 - r)$ is called the **sum to infinity**.

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Worked Problems on Geometric Progressions and Exercise 30

- **Problem 13.** Which term of the series 2187, 729, 243, ... is $1/9$?
[$1/9$ is the 10th term of the GP]
- **Exercise 6.** Find the sum to infinity of the series $2\frac{1}{2}, -1\frac{1}{4}, 5/8, \dots$
[5/3]

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Further Worked Problems on Geometric Progressions

- **Problem 16.** In a geometric progression the 6th term is 8 times the 3rd term and the sum of the 7th and 8th terms is 192. Determine (a) the common ratio, (b) the 1st term, and (c) the sum of the 5th to 11th terms, inclusive.
[(a) $r = 2$ (b) $a = 1$ (c) 2032]
- **Problem 18.** If £100 is invested at compound interest of 8% per annum, determine (a) the value after 10 years, (b) the time, correct to the nearest year, it takes to reach more than £300.
[(a) £215.89 (b) 15 years]

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Exercise 31

- **Exercise 6.** If £250 is invested at compound interest of 6% per annum determine (a) the value after 15 years, (b) the time, correct to the nearest year, it takes to reach £750.
[(a) £599.14 (b) 19 years]
- **Exercise 7.** A drilling machine is to have 8 speeds ranging from 100 rev/min to 1000 rev/min. If the speeds form a geometric progression determine their values, each correct to the nearest whole number.
[100, 139, 193, 268, 373, 518, 720, 1000 rev/min]

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