Chapter 3: Partial Fractions

謝仁偉 助理教授
jenwei@mail.ntust.edu.tw
國立台灣科技大學 資訊工程系
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Outline

- Introduction to Partial Fractions
- Linear Factors
- Repeated Linear Factors
- Quadratic Factors

Introduction to Partial Fractions (1/2)

- By algebraic addition,

$$\frac{1}{x-2}+\frac{3}{x+1}=\frac{(x+1)+3(x-2)}{(x-2)(x+1)}=\frac{4x-5}{x^2-x-2}$$

The reverse process of moving from $\frac{4x-5}{x^2-x-2}$ to $\frac{1}{x-2}+\frac{3}{x+1}$ is called resolving into partial fractions.

Introduction to Partial Fractions (2/2)

- In order to resolve an algebraic expression into partial fractions:
  1. The **denominator must factorize** and
  2. The **numerator must be at least one degree less than the denominator**

Three Types of Partial Fractions

Type	Denominator containing	Expression	Form of partial function
1	Linear factors (see Problems 1 to 4)	$\frac{f(x)}{(x+a)(x-b)(x+c)}$	$\frac{A}{(x+a)}+\frac{B}{(x-b)}+\frac{C}{(x+c)}$
2	Repeated linear factors (see Problems 5 to 7)	$\frac{f(x)}{(x+a)^3}$	$\frac{A}{(x+a)}+\frac{B}{(x+a)^2}+\frac{C}{(x+a)^3}$
3	Quadratic factors (see Problems 8 to 9)	$\frac{f(x)}{(ax^2+bx+c)(x+d)}$	$\frac{Ax+B}{(ax^2+bx+c)}+\frac{C}{(x+d)}$

- $f(x)$ is assumed to be of less degree than the relevant denominator
- A, B, and C are constant to be determined.

Linear Factors (1/2)

- **Problem 1.** Resolve $(11 - 3x)/(x^2 + 2x - 3)$ into partial fractions.
  [To determine constants A and B, values of x are chosen to make the term in A or B equal to zero.]
  [2/(x – 1) – 5/(x + 3)]

- **Problem 2.** Convert $\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$ into the sum of three partial fractions.
  [4/(x + 1) – 3/(x – 2) + 1/(x + 3)]

Linear Factors (2/2)

- **Problem 4.** Express $\frac{x^3-2x^2-4x-4}{x^2+x-2}$ in partial fractions.
  [x – 3 + 4/(x + 2) – 3/(x – 1)]

Exercise 13

Resolve the following into partial fractions.

- **Exercise 3.** $(x^2 – 3x + 6)/[x(x – 2)(x – 1)]$
  [3/x + 2/(x – 2) – 4/(x – 1)]
- **Exercise 5.** $(x^2 + 9x + 8)/(x^2 + x – 6)$
  [1 + 2/(x + 3) + 6/(x – 2)]
- **Exercise 7.** $(3x^3 – 2x^2 – 16x + 20)/[(x – 2)(x + 2)]$
  [3x – 2 + 1/(x – 2) – 5/(x + 2)]

Repeated Linear Factors

- **Problem 6.** Express $\frac{5x^2-2x-19}{(x+3)(x-1)^2}$ as the sum of three partial fractions.
  [2/(x + 3) + 3/(x – 1) – 4/(x – 1)²]

- **Problem 7.** Resolve (3x² + 16x + 15)/(x + 3)³ into partial fractions.
  [3/(x + 3) – 2/(x + 3)² – 6/(x + 3)³]

Exercise 14

- **Exercise 2.** (x² + 7x + 3)/[x²(x + 3)]
  [1/x² + 2/x – 1/(x + 3)]

- **Exercise 3.** (5x² – 30x + 44)/(x – 2)³
  [5/(x – 2) – 10/(x – 2)² + 4/(x – 2)³]

Quadratic Factors & Exercise 15

- **Problem 8.** Express $\frac{7x^2+5x+13}{(x^2+2)(x+1)}$ in partial fractions.
  [(2x + 3)/(x² + 2) + 5/(x + 1)]

- **Exercise 3.** (15 + 5x + 5x² – 4x³)/[x²(x² + 5)]
  [1/x + 3/x² + (2 – 5x)/(x² + 5)]
- **Exercise 4.** (x³ + 4x² + 20x – 7)/[(x – 1)²(x² + 8)]
  [3/(x – 1) + 2/(x – 1)² + (1 – 2x)/(x² + 8)]