

Chapter 2: Inequalities

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Outline

- Inequalities Involving a Modulus
- Inequalities Involving Quotients
- Inequalities Involving Square Functions
- Quadratic Inequalities

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Inequalities Involving a Modulus

- The **modulus** (絕對值) of a number is the size of the number, regardless of sign.
 - $|t| < 1$ means $-1 < t < 1$
 - $|x| > 3$ means $x > 3$ or $x < -3$
- **Problem 4.** Solve the following inequality:
 $|3x + 1| < 4$
 $[-5/3 < x < 1]$
- **Problem 6.** Solve the inequality: $|3z - 4| > 2$
 $[z > 2 \text{ or } z < 2/3]$

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Exercise 9

Solve the following inequalities:

- **Exercise 3.** $|2x - 1| < 4$
 $[-3/2 < x < 5/2]$
- **Exercise 4.** $|3t - 5| > 4$
 $[t > 3 \text{ or } t < 1/3]$

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Inequalities Involving Quotients

- If $p/q > 0$ then p/q must be a positive, either
 - Both p and q are positive or
 - Both p and q are negative
- If $p/q < 0$ then p/q must be a negative, either
 - p is positive and q is negative or
 - p is negative and q is positive

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Problems & Exercise 10

- **Problem 7.** Solve the inequality: $\frac{t+1}{3t-6} > 0$
[$t > 2$ or $t < -1$]
- **Problem 8.** Solve the inequality: $\frac{2x+3}{x+2} \leq 1$
[$-2 < x \leq -1$]
- **Exercise 2.** Solve the inequality: $\frac{2t+4}{t-5} > 1$
[$t > 5$ or $t < -9$]
- **Exercise 3.** Solve the inequality: $\frac{3z-4}{z+5} \leq 2$
[$-5 < z \leq 14$]

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Inequalities Involving Square Functions

- The following two general rules apply when inequalities involve square functions:
 - If $x^2 > k$ then $x > \sqrt{k}$ or $x < -\sqrt{k}$
 - If $x^2 < k$ then $-\sqrt{k} < x < \sqrt{k}$
- **Problem 9.** Solve the inequality: $t^2 > 9$
[$t > 3$ or $t < -3$]
- **Problem 12.** Solve the inequality: $t^2 < 9$
[$-3 < t < 3$]

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Exercise 11

- **Exercise 4.** $3k^2 - 2 \leq 10$
[$-2 \leq k \leq 2$]
- **Exercise 8.** $(4k + 5)^2 > 9$
[$k > -1/2$ or $k < -2$]

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Quadratic Inequalities

- Inequalities involving quadratic expressions are solved using either
 - Factorization or
 - Completing the square
$$x^2 + bx + c \equiv (x + b/2)^2 + c - (b/2)^2$$
- **Problem 15.** Solve the inequality: $x^2 + 2x - 3 > 0$
[$x > 1$ or $x < -3$]
- **Problem 17.** Solve the inequality: $x^2 + 6x + 3 < 0$
[$(-\sqrt{6} - 3) < x < (\sqrt{6} - 3)$]

Exercise 12

Solve the following inequalities:

- **Exercise 1.** $x^2 - x - 6 > 0$
[$x > 3$ or $x < -2$]
- **Exercise 2.** $t^2 + 2t - 8 \leq 0$
[$-4 \leq t \leq 2$]
- **Exercise 5.** $z^2 + 4z + 4 \leq 4$
[$-4 \leq z \leq 0$]