Chapter 1: Algebra

謝仁偉 助理教授
jenwei@mail.ntust.edu.tw
國立台灣科技大學 資訊工程系
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Outline

- Basic Operations and Laws of Indices
- Transposition of Formulae
- Polynomial Division
- The Remainder Theorem

Basic Operations and Laws of Indices

(i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{m \times n}$ (iv) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

(v) $a^{-n} = \frac{1}{a^n}$ (vi) $a^0 = 1$

Problems & Exercise 1

- **Problem 3.** Simplify $a^3b^2c^4/abc^{-2}$ and evaluate when a = 3, b = 1/8, and c = 2.
  [a^2bc^6, 72]
- **Problem 5.** Simplify $\frac{(x^2\sqrt{y})(\sqrt{x}\sqrt[3]{y^2})}{(x^5y^3)^{1/2}}$
  [$y^{-1/3}$ or $1/y^{1/3}$]
- **Exercise 8.** Simplify $\frac{(a^3b^{1/2}c^{-1/2})(ab)^{1/3}}{(\sqrt{a^3}\sqrt{b}\,c)}$
  [$a^{11/6}b^{1/3}c^{-3/2}$]

Transposition of Formulae

- **Problem 15.** Transpose the formula $v = u + \frac{ft}{m}$ to make f the subject.
  [f = m(v – u)/t]

- **Problem 17.** Given that $\frac{D}{d} = \sqrt{\frac{f+p}{f-p}}$, express p in terms of D, d, and f.
  [p = f(D² – d²)/(d² + D²)]

Exercise 3

- **Exercise 6.** Make l the subject of $t = 2p\sqrt{l/g}$
  [l = t²g/4p²]

- **Exercise 7.** Transpose $m = uL/(L + rCR)$ for L.
  [L = mrCR/(u – m)]

Polynomial Division & Exercise 5

- **Problem 24.** Divide $3x^3 + x^2 + 3x + 5$ by $x + 1$.
  [$3x^2 - 2x + 5$]

- **Problem 25.** Simplify $(x^3 + y^3)/(x + y)$.
  [$x^2 - xy + y^2$]

- **Exercise 8.** Determine $(5x^4 + 3x^3 – 2x + 1)/(x – 3)$.
  [$5x^3 + 18x^2 + 54x + 160 + 481/(x - 3)$]

The Remainder Theorem & Exercise 7

- If $(ax^2 + bx + c)$ is divided by $(x – p)$, the remainder will be $ap^2 + bp + c$.
- If $(ax^3 + bx^2 + cx + d)$ is divided by $(x – p)$, the remainder will be $ap^3 + bp^2 + cp + d$.
- **Problem 32.** Determine the remainder when $(x^3 – 2x^2 – 5x + 6)$ is divided by (a) $(x – 1)$ and (b) $(x + 2)$. Hence factorize the cubic expression.
  [$(x - 1)(x + 2)(x - 3)$]
- **Exercise 5.** Determine the value of ‘a’ if $(x + 2)$ is a factor of $(x^3 – ax^2 + 7x + 10)$. [a = –3]