離散數學第一次小考 (範圍:ch1 & ch2) 授課老師：鍾國亮教授

日期:2010/10/4(一)

1. (10 points) What is the number of nonnegative integer solutions of $x\_{1}+x\_{2}+x\_{3}\leq 15$ ($x\_{1},x\_{2},x\_{3}\geq 0$)?
2. (10 points) Cynthia has 9 classmates in a computer programming class. For the final project as teamwork, in how many ways can she team up with two or more of them?
3. (10 points) Solve

, n is even.

1. (10 points) Let Ax = {a, b, c, d, e}, Px ={1/2, 1/4, 1/8, 1/16, 1/16}, and consider the code *C*(a) = 0, *C*(b) = 10, *C*(c) = 110, *C*(d) = 1110, C(e) = 1111. Please find the expected length *L(C)* of this code.
2. (10 points) Derive the expectation value and the variance of the Bernoulli distribution.

離散數學第一次小考答案 (範圍:ch1 & ch2)

日期:99/10/4

1. 令 $x\_{4}\geq 0$,

則$x\_{1}+x\_{2}+x\_{3}+x\_{4}=15$

共有$\left(\genfrac{}{}{0pt}{}{15+4-1}{4-1}\right)=\left(\genfrac{}{}{0pt}{}{18}{3}\right)=\frac{18∙17∙16}{3∙2∙1}$ = 816種

1. Cynthia至少和兩位同學一組的方法數有:$\left(\genfrac{}{}{0pt}{}{9}{2}\right)+\left(\genfrac{}{}{0pt}{}{9}{3}\right)+\left(\genfrac{}{}{0pt}{}{9}{4}\right)+\left(\genfrac{}{}{0pt}{}{9}{5}\right)+\left(\genfrac{}{}{0pt}{}{9}{6}\right)+\left(\genfrac{}{}{0pt}{}{9}{7}\right)+\left(\genfrac{}{}{0pt}{}{9}{8}\right)+\left(\genfrac{}{}{0pt}{}{9}{9}\right)=2^{9}-\left(\genfrac{}{}{0pt}{}{9}{0}\right)-\left(\genfrac{}{}{0pt}{}{9}{1}\right)$ = 512 - 1 - 9 = 502種
2.  (當n為偶數)
3. 令l(i)為c(i)的長度及p(i)為i出現的機率,i=a,b,c,d,e

則l(a)=1, l(b)=2, l(c)=3, l(d)=4, l(e)=4

 P(a)=$\frac{1}{2} ,$ P(b)=$\frac{1 }{4} , $ P(c)=$\frac{1}{8} ,$ P(d)=$\frac{1}{16} ,$ P(e)=$\frac{1}{16}$

故編碼長度*L(C)*的期望值:

*L(C)*

=$l(a)∙P(a)$+$ l(b)∙P(b)$+$ l(c)∙P(c)$+$ l(d)∙P(d)$+$ l(e)∙P(e)$

=

5. 從伯努利分佈的介紹中, 伯努利分佈被定義為：



根據上式,伯努利分佈的期望值為：



根據變異數的定義, 伯努利分佈的變異數為



求得伯努利分佈的變異數後, 根據標準差的定義, 很自然可得到伯努利分佈的標準差為

