Characterization of the Output of an ATM Output Buffer Receiving Self-Similar Traffic

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Abstract— **We investigate the departure process of an ATM output buffer intaking asymptotically second-order self-similar traffic using matrix-analytic technique. The results obtained include the marginal and joint distribution of interdeparture times. We show that output and input processes asymptotically possess the same tail and smoothing effect takes place only for traffic with small Hurst parameters.**

I. INTRODUCTION

In the past, self-similarity has mainly been exploited in non-communication areas, such as hydrology, geophysics, economics, mathematics, and statistics. Mandelbrot [1] was first to apply this concept to communication systems. But the application of self-similarity to communications is still in its infancy. In a recent article, Leland, Taqqu, Willinger, and Wilson [2] showed that self-similarity appears in the aggregated traffic across high-resolution Ethernet local area networks (LAN's). The self-similar behaviors of traffic over wide area networks (WAN's) [3], and other working networks, e.g., variable bit rate (VBR) video over asynchronous transfer mode (ATM) [4], have also been studied.

The works we mentioned in the above all point out that self-similar or fractal behavior in traffic over contemporary communication networks can not be captured using traditional traffic models such as Poisson, Markov-modulated Poisson, and fluid flow model. Therefore, a new class of traffic models and a new round of performance analysis necessitate. Research in this regard has been reported recently, e.g., [5]–[7]. Diamond and Alfa [7] showed that the renewal process with (asymptotic) hyperbolically distributed interarrival time exhibits self-similar behavior and analyzed the resulting hyper/D/1 queue using the matrix-analytic technique to obtain the mean and standard deviation of queue length. Following [7], we use the heavy-tailed renewal process to capture self-similarity.

Among various transport technologies, ATM is promising for interconnection of enterprise networks composed of several LAN's carrying perhaps VBR traffic. Although a few works have addressed the node-level performance issues of ATM queues when self-similar traffic is taken into consideration, performance analysis should not limit to node level only since ATM networks are connection-oriented. One way to extend nodal analysis to connection-wise analysis is first to analyze the departure process, then recursively perform nodal analysis. So far, only very few papers have attempted to address the issue of output process characterization pertinent to self-similar input traffic. Vamvakos and Anantharam [8] studied the departure process of a leaky bucket system, one of the schemes proposed for flow control in ATM networks, receiving long-range dependent input traffic. They established upper and lower bounds for the covariance sequence of the output counting process and concluded that long-range dependence (LRD) cannot be removed through the leaky bucket scheme. But the structure of the output process has not been provided. In this paper, we characterize the output process of an ATM queue that receives asymptotically second-order self-similar traffic. The characterization is done through finding the interdeparture time distribution and joint distribution of successive interdeparture times. We show that the tail of the output process asymptotically remains the same as that of the input process.

The rest of the paper is organized as follows. Section II gives the queueing model we employed and introduces the related notation. In Section III, we analyze the departure process of the corresponding queueing system. Results obtained include the interdeparture time distribution and the joint distribution of successive interdeparture times. Section IV gives numerical examples and discussions. Finally, Section V concludes the paper.

II. QUEUEING MODEL AND NOTATION

We consider an ATM switch with n input/output ports in this paper. For a specific output buffer, traffic streams from various input ports are served in a round-robin manner and each receives a service time of T_s. To properly reflect the working mechanism, we employ the discrete-time model in which time is discretized into basic time unit with duration T_s/n. Each input port may or may not generate a cell during each time unit. Here, we assume infinite buffers and the aggregated cell streams to a specific output buffer follow a heavy-tailed renewal process with interarrival time distribution $f(i)$. In the following, we use $f_i = f(i)$ for convenience. Due to the renewal property, embedded Markov chain (EMC) at arrival epochs can be employed to facilitate analysis. For convenience, let N_i and R_i, respectively, denote the queue length seen by the ith cell on its arrival and the remaining service time

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of the cell in service seen by the ith cell. Clearly, the EMC at arrival epochs has the state space $S = \{(j,k)|N_i = j, R_i = k; j = 0,1,\ldots; k = 1,\ldots,n \text{ or } j = k = 0\}$. Note that (0,0) denotes the state that system is idle. The transition probability matrix of the EMC at arrival epochs has the following form [7]:

$$\boldsymbol{P} = \begin{bmatrix} B_{00} & \boldsymbol{B}_{01} & \boldsymbol{0} & \boldsymbol{0} & \cdots \\ \boldsymbol{B}_{10} & \boldsymbol{B}_{11} & \boldsymbol{A}_0 & \boldsymbol{0} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix}, \tag{1}$$

where

$$\boldsymbol{A}_0 = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ f_1 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ f_{n-1} & f_{n-2} & \cdots & f_1 & 0 \end{bmatrix},$$
$$\boldsymbol{A}_i = \begin{bmatrix} f_{in} & f_{in-1} & \cdots & f_{(i-1)n+1} \\ f_{in+1} & f_{in} & \cdots & f_{(i-1)n+2} \\ \vdots & \vdots & \ddots & \vdots \\ f_{(i+1)n-1} & f_{(i+1)n-2} & \cdots & f_{in} \end{bmatrix}, \; i \geq 1, \tag{2}$$

and

$$\begin{aligned} B_{00} &= F_n, \\ \boldsymbol{B}_{01} &= \begin{bmatrix} f_{n-1} & \ldots & f_1, 0 \end{bmatrix}, \\ \boldsymbol{B}_{i0} &= \begin{bmatrix} F_{in+1} & \ldots & F_{(i+1)n} \end{bmatrix}^T, \\ \boldsymbol{B}_{i1} &= \boldsymbol{A}_i, \; i \geq 1, \end{aligned} \tag{3}$$

with $F_i = \sum_{j=i}^{\infty} f_j$ and the operator ^{T} indicates the transpose of a matrix. The stationary transition probability vector $\boldsymbol{x}$ of $\boldsymbol{P}$, i.e., $\boldsymbol{xP} = \boldsymbol{x}$, can be partitioned into $\boldsymbol{x} = (x_{0,0}, \boldsymbol{x}_0, \boldsymbol{x}_1, \ldots)$ with $\boldsymbol{x}_j = (x_{j,1}, x_{j,2}, \ldots, x_{j,n})$, $j \geq 0$, where $x_{j,k} = \lim_{i\to\infty} Pr\{N_i = j, R_i = k\}$. The solution is in the form of $\boldsymbol{x} = (x_{0,0}, \boldsymbol{x}_0, \boldsymbol{x}_0\boldsymbol{R}, \ldots)$, where $\boldsymbol{R} = \sum_{l=0}^{\infty} \boldsymbol{R}^l \boldsymbol{A}_l$ [9]. Ramaswami [10] proposed an efficient algorithm to solve the nonlinear matrix equation $\boldsymbol{R} = \sum_{l=0}^{\infty} \boldsymbol{R}^l \boldsymbol{A}_l$.

III. Output Process Characterization

A. Interdeparture Time Distribution

Define $T_{A,i}$ and $T_{D,i}$, respectively, to be the interarrival time between the ith and the $(i+1)$st cell arrivals and the interdeparture time between the ith and the $(i+1)$st cell departures. Let $\tilde{d}(m) = Pr\{T_{D,i} = m\}$ $(m \geq n)$ denote the interdeparture time distribution. Then $\tilde{d}(m)$ can be obtained using system time diagrams for the following two cases, i.e., $m = n$ and $m \geq n+1$.

$$\tilde{d}(m) = \begin{cases} \displaystyle\sum_{l=1}^{n} f_l x_{0,0} + \sum_{j=0}^{\infty}\sum_{k=1}^{n}\sum_{l=1}^{(j+1)n+k} f_l x_{j,k}, & \text{for } m = n, \\ \displaystyle f_m x_{0,0} + \sum_{j=0}^{\infty}\sum_{k=1}^{n} f_{jn+k+m} x_{j,k},, & \text{for } m \geq n+1. \end{cases} \tag{4}$$

From (4), we can easily derive

$$\tilde{d}(m) \sim f(m), \text{ as } m \to \infty \tag{5}$$

where $p(x) \sim q(x)$ means $[p(x)/q(x)] \to 1$ as $x \to \infty$. This tells us that the tail of the output process is asymptotically the same as that of the input process.

B. Joint Distribution of Successive Interdeparture Times

Now, we derive the joint distribution of m (≥ 2) successive interdeparture times. Let $\tilde{d}(d_1, d_2, \ldots, d_m)$ denote the joint distribution of m successive interdeparture times, i.e., $\tilde{d}(d_1, d_2, \ldots, d_m) = Pr\{T_{D,i} = d_1, T_{D,i+1} = d_2, \ldots, T_{D,i+m-1} = d_m\}$ $(d_i \geq n, \forall i)$. Using system time diagrams, it can easily be shown that $\tilde{d}(d_1, d_2, \ldots, d_m)$ for $d_i > n, \forall i$, turns out to be

$$\tilde{d}(d_1, d_2, \ldots, d_m) = \tilde{d}(d_1) \prod_{i=2}^{m} f_{d_i}. \tag{6}$$

To derive more general result, we further introduce L_t's $(t \geq 1)$ as follows:

$$L_t = \begin{cases} \{l_t | 0 \leq l_t \leq (j_t+1)n + k_t\}, & \text{if } d_t = n, \\ \{l_t | l_t = 0\}, & \text{if } d_t > n \end{cases} \tag{7}$$

where $l_t \equiv max\{0, T_{D,i+t-1} - T_{A,i+t}\}$ represents the time between the $(i+t)$th cell arrival and the $(i+t-1)$st cell departure if the $(i+t-1)$st cell departure occurs behind the $(i+t)$th cell arrival; otherwise it is set to zero. State (j_t, k_t) represents the state seen by the $(i+t-1)$st cell on its arrival and j_t, k_t for $t \geq 2$ can be derived via $j_t = \lfloor \frac{l_{t-1}}{n} \rfloor$, $k_t = l_{t-1} - \lfloor \frac{l_{t-1}}{n} \rfloor n = l_{t-1} - j_t n$ for $l_{t-1} \in L_{t-1}$. Then L_t's $(t \geq 2)$ can also be expressed as $\{l_t | 0 \leq l_t \leq l_{t-1} + n\}$ when $d_t = n$.

By definition, $(j_t n + k_t + d_t - l_t)$ denotes the interarrival time between the $(i+t-1)$st cell arrival and the $(i+t)$th cell arrival, see Fig. 1. Then $\tilde{d}(d_1, d_2, \ldots, d_m)$ can be written as

$$\begin{aligned} &\tilde{d}(d_1, d_2, \ldots, d_m) \\ &\quad = \sum_{(j_1,k_1)\in S}\sum_{l_1\in L_1} f_{j_1 n + k_1 + d_1 - l_1} x_{j_1,k_1} \\ &\qquad \times \prod_{t=2}^{m}\sum_{l_t\in L_t} f_{j_t n + k_t + d_t - l_t} \\ &\quad = \sum_{(j_1,k_1)\in S}\sum_{l_1\in L_1} f_{j_1 n + k_1 + d_1 - l_1} x_{j_1,k_1} \\ &\qquad \times \prod_{t=2}^{m}\sum_{l_t\in L_t} f_{l_{t-1} + d_t - l_t}. \end{aligned} \tag{8}$$

The first equality results from the renewal property of the arrival process and the second equality results from the fact that $j_t n + k_t = l_{t-1}$. Note that (8) indeed reduces to (6) when the case $d_i > n, \forall i$, is considered since $l_t = 0, \forall t$. This checks the correctness of (8).

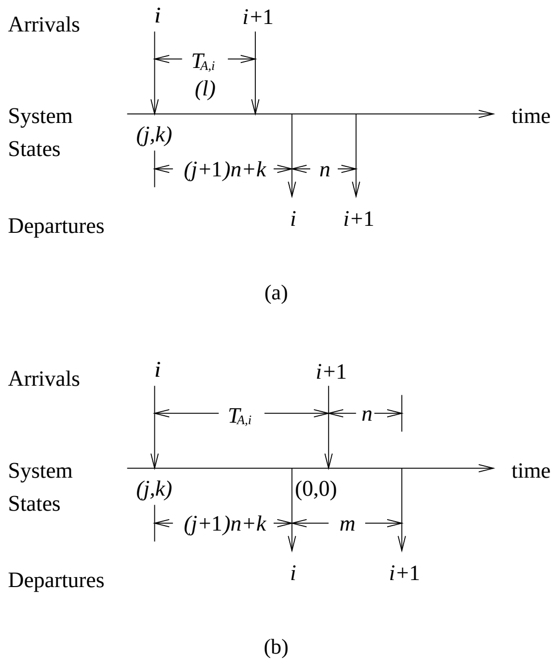

Fig. 1. System time diagrams: (a) the case $T_{D,i} = n$ that yields $T_{A,i} = l$ $(1 \le l \le (j+1)n + k)$; (b) the case $T_{D,i} = m$ $(> n)$ that yields $T_{A,i} = jn + k + m$.

TABLE I

TRAFFIC LOAD UNDER DIFFERENT PARAMETER SETS.

(a, θ)	(15,1.4)	(15,1.6)	(15,1.8)
Traffic load	0.5850	0.7620	0.9031
(a, θ)	(45,1.4)	(45,1.6)	(45,1.8)
Traffic load	0.1938	0.2519	0.2981

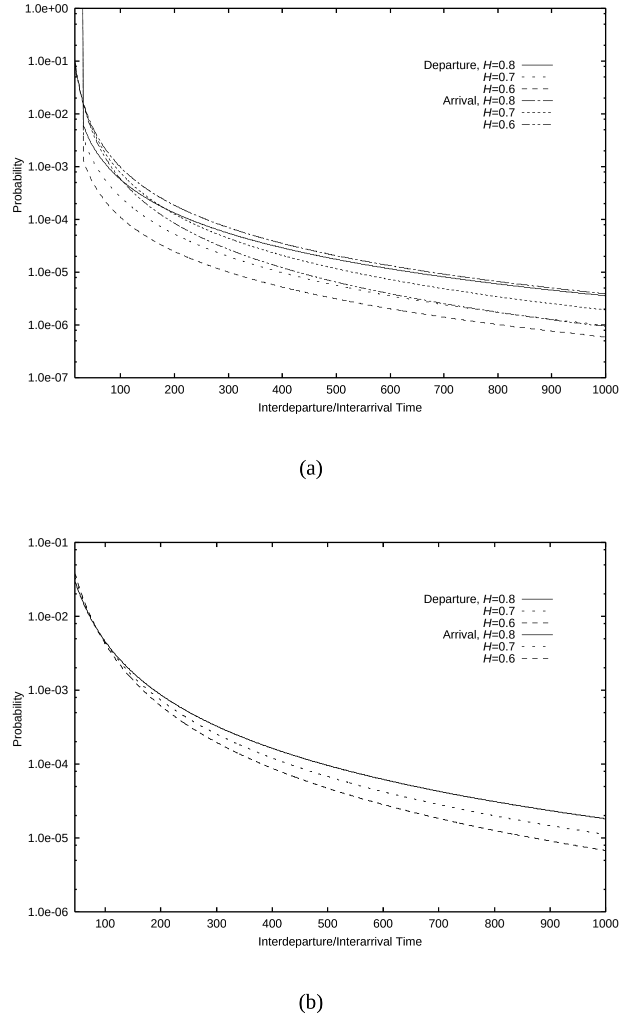

Fig. 2. Comparison between interdeparture and interarrival time distributions for traffic streams with different Hurst parameters: (a) $a = 15$ and (b) $a = 45$ under $T_s = n = 30$.

IV. Numerical Examples and Discussions

Consider the renewal arrival process with the discrete Pareto type interarrival time distribution [3], [7], i.e.,

$$F_i = \begin{cases} (\frac{a}{i})^{\theta}, & a, \theta \ge 0, i \ge a, \\ 1, & \text{otherwise,} \end{cases} \tag{9}$$

and

$$f_i = \begin{cases} (\frac{a}{i})^{\theta} - (\frac{a}{i+1})^{\theta}, & a, \theta \ge 0, i \ge a, \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

where a represents the location parameter and θ is the shape parameter. According to the definition given by [3], it can easily be shown that the discrete Pareto distribution is heavy-tailed. Then the arrival process exhibits asymptotically second-order self-similar nature mentioned earlier. The corresponding Hurst parameter is related to the shape parameter via $H = (3 - \theta)/2$ [7]. For self-similar traffic, the Hurst parameter falls within $0.5 < H < 1$ and it governs the degree of burstiness. Traffic is more bursty if it has a larger Hurst parameter.

Now, let service time be 30 time units (i.e., $T_s = n = 30$). As for parameters of input traffic, we set $a = 15$, 45 and vary θ from 1.4 to 1.8 to represent different degrees of the self-similarity (corresponding to Hurst parameter H varying from 0.8 to 0.6). The values of traffic load under different parameters are listed in Table I. Figs. 2(a) and 2(b) show the comparison between the distributions of both interarrival and interdeparture times under $a = 15$ and $a = 45$, respectively. Fig. 2(a) reveals the followings: 1) Interdeparture time distributions are intensified at 30 (the value of service time), and decay faster than interarrival time distributions over short range. Traffic with a larger Hurst parameter incurs a slower decay, i.e., smoothing effect takes place only for traffic with small Hurst parameters. 2) The tail of the interdeparture time distribution asymptotically remains the same as that of the interarrival distribution, i.e., both input and output processes are heavy-tailed. This fact has also been concluded from (5). Fig. 2(b) shows that the interdeparture time distribution is the same as the interarrival time distribution. This is due to that interarrival times are not smaller than 45 (larger than 30) time units when the location parameter a is fixed at 45. Of course, both input and output processes are heavy-tailed.

V. Conclusions

The impact of self-similarity on system performance in contemporary telecommunication networks, e.g., ATM networks and Ethernet LAN's, has been noticed in the literature. Although traffic modeling and performance analysis pertinent to self-similarity have been intensively studied, works on the output process characterization are so far rare. Vamvakos and Anatharam [8] studied the departure process of a leaky bucket system when input is the long-range dependent traffic. Although the upper and lower bounds of the covariance sequence of the output counting process were obtained and the phenomenon that LRD cannot be removed through the leaky bucket system was observed, no further insight of the output process is investigated. In this paper, we have analyzed the output process of an ATM switch output buffer via obtaining the interdeparture time distribution and joint distribution of successive interdeparture times. We show that the smoothing effect takes place only for traffic with small Hurst parameters and that the tail of output distribution is asymptotically the same as that of input distribution.

Once the output process characterization is completed, it can be applied to extend the nodal performance analysis to connection-wise performance analysis through moment matching schemes in a recursive manner. This is a topic of future research.

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