Cellular Digital Packet Data: Channel Availability

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***Abstract*—Cellular digital packet data (CDPD) networks borrow idle radio channels from advanced mobile phone standard (AMPS) cellular networks to send packet data traffic. We explore the amount of unused AMPS capacity available to parasitic data networks such as CDPD. Due to trunking inefficiencies, a relatively large amount of spare AMPS airlink capacity can be found. In addition, the periods of time AMPS channels are idle are fairly long. Due to fluctuations in the number of AMPS channels in a sector busy serving AMPS calls, there will be periods of time when there is not an ample number of idle AMPS channels for CDPD. Resource contention between CDPD and AMPS will affect the amount of data that can be carried on the CDPD airlink and the performance of higher layer protocols. We present analytical results which serve as the basis for cell site capacity engineering rules for CDPD networks.**

***Index Terms*— AMPS, CDPD, resource contention, wireless data.**

I. Introduction

CELLULAR systems using the frequency-division multiple access-based advanced mobile phone standard (AMPS) are pervasive in North America and many other parts of the world. Due to the relatively small number of AMPS channels in a "typical" AMPS sector and the desire of AMPS service providers to keep call-blocking probabilities tolerable, utilization of AMPS channels in a sector, out of necessity, is kept fairly low.

Enter cellular digital packet data (CDPD) [1]. One of the premises of CDPD systems is that there is a relatively large number of idle AMPS channels on average that can be used to carry short bursts of packet data traffic.[1]

In this paper, we quantify the amount of unused AMPS air time available to a parasitic wireless data network such as CDPD. We find that even at peak AMPS call loads, a relatively large amount of spare AMPS airlink capacity can be found in many AMPS systems. Furthermore, we find that the periods of time AMPS channels are idle are of sufficient duration to be usable by a packet-switched data service such as CDPD.

Utilization of unused AMPS capacity by CDPD networks comes at the cost of an increase in the amount of ambient interference in the AMPS frequency bands. The impact of CDPD transmissions on interference levels observed by AMPS calls is quantified in [4], [10], and [11]. Analysis in [10] shows that CDPD may not be appropriate for large (54 AMPS channels/cell) omni cells with seven-cell reuse because of interference constraints. As discussed in [10], AMPS coverage with a seven-cell reuse pattern is inadequate without sectorization, even for voice alone. For this reason, many cellular service providers deploy AMPS systems using seven-cell reuse with three-sector cells. An analysis in [11] shows that CDPD does not cause debilitating levels of interference in AMPS systems employing this popular reuse scheme. Throughout the remainder of this paper, we make the implicit assumption that CDPD transmissions do not cause debilitating levels of interference to AMPS calls. Our focus instead is the impact of AMPS call loads and other system parameters on the amount of CDPD traffic that can be carried in an AMPS sector as well as other CDPD-related performance measures.

Due to the stochastic nature of AMPS call attempts and completions, the amount of AMPS airlink capacity available to CDPD fluctuates. We explore how efficiently CDPD exploits unused AMPS airlink capacity in the presence of this fluctuation. In particular, there will be periods of time when there will not be an ample number of idle AMPS channels to satisfy the demands of CDPD. Resource contention between AMPS and CDPD during these periods will affect the amount of data that can be carried over CDPD airlinks and the performance of higher-layer protocols. Study of these resource contention issues provides a basis for CDPD capacity engineering rules.

II. How Much Idle AMPS Air Time is There?

Cellular service providers typically equip enough channels at AMPS cell sites so that the probability that all channels in a sector are occupied when a subscriber attempts a call is below a threshold-blocking probability, p_{block}. Peak-blocking probabilities in the 1%–3% range are typical for mature systems, but may reach higher levels in underengineered, hot-spot cells.

We assume that equal treatment is given to "fresh" calls and calls that arrive at a cell due to a handoff; no AMPS channels are expressly reserved for handoffs. We assume further that AMPS call attempts and handoffs in a sector are generated in accordance with a Poisson[2] process of rate λ. AMPS calls that find all AMPS channels in a sector busy are cleared (no retrials). We assume that the length of time an AMPS channel is occupied by an AMPS call, called the *AMPS channel-holding time*, is generally distributed with mean $\overline{T}$. The term $a = \lambda\overline{T}$ denotes the offered AMPS call load in the sector in Erlangs. The Erlang B formula [9], $B(c, a)$, [shown in (1)] gives the steady-state blocking probability observed by AMPS calls generated in a sector with a total of c AMPS channels

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[1] The CDPD system specification also has provisions which allow CDPD dedicated use of an AMPS channel if desired.

[2] As shown in [3], the Poisson assumption may not adequately capture mobility-dependent characteristics of call handoffs. The assumption is necessary for analytical tractability of the model, however.

and an offered call load of a Erlangs

$$B(c, a) = \frac{\dfrac{a^c}{c!}}{\displaystyle\sum_{j=0}^{c} \frac{a^j}{j!}}. \tag{1}$$

Let $a_{\max}(c)$ denote the maximum AMPS call load that a sector equipped with c AMPS channels can support while still satisfying the maximum tolerable blocking-probability requirement p_{block}. That is

$$B[c, a_{\max}(c)] = p_{\text{block}}. \tag{2}$$

Assume that AMPS calls are assigned to AMPS channels in such a way that over time, on average, each channel receives an equal fraction of the AMPS call load. This assumption is true of the common channel-selection algorithms used by AMPS equipment vendors, including "round robin" and "route to the most idle channel." Under this assumption we calculate $\rho(c, a)$, the fraction of time each AMPS channel in a sector is occupied by an AMPS call

$$\rho(c, a) = \frac{a[1 - B(c, a)]}{c}. \tag{3}$$

Let $\overline{I}(c, a)$ denote the average length of time each AMPS channel in the sector is idle

$$\frac{\overline{T}}{\overline{I}(c, a) + \overline{T}} = \rho(c, a) \tag{4}$$

and hence

$$\overline{I}(c, a) = \frac{1 - \rho(c, a)}{\rho(c, a)} \overline{T}. \tag{5}$$

Another quantity of interest is $\overline{n}_{idle}(c, a)$, the average number of AMPS channels in the sector that is not occupied by an AMPS call where

$$\overline{n}_{idle}(c, a) = c - a[1 - B(c, a)]. \tag{6}$$

Fig. 1 shows plots of the normalized ($\overline{T} = 1$) average idle time per AMPS channel $\overline{I}[c, a_{\max}(c)]$ versus the number of AMPS channels in a sector for several blocking probabilities. It is important to note that as the number of AMPS channels in a sector increases and blocking probability is held constant, the average length of time each AMPS channel is idle decreases. This is a restatement of a well-known result on the efficiency of large trunk groups: at identical call-blocking probabilities, large trunk groups are capable of carrying more calls per trunk than smaller trunk groups (see [16]). Hence, the length of time an AMPS channel can be used to send data before the channel is needed again by AMPS decreases as the number of AMPS channels per sector increases.

Fig. 2 shows plots of the average number of idle AMPS channels per sector as a function of the number of AMPS channels in a sector for several AMPS call-blocking probabilities. Note that the curves are strictly concave in the number of AMPS channels per sector—another well-known property of the Erlang B loss function [16].

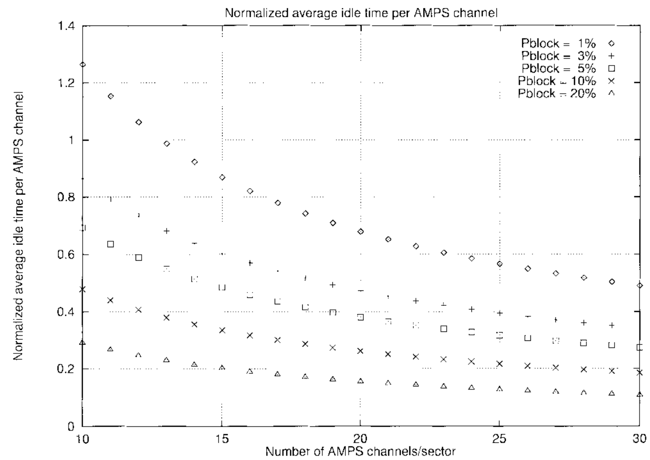

Fig. 1. Average idle time per AMPS channel.

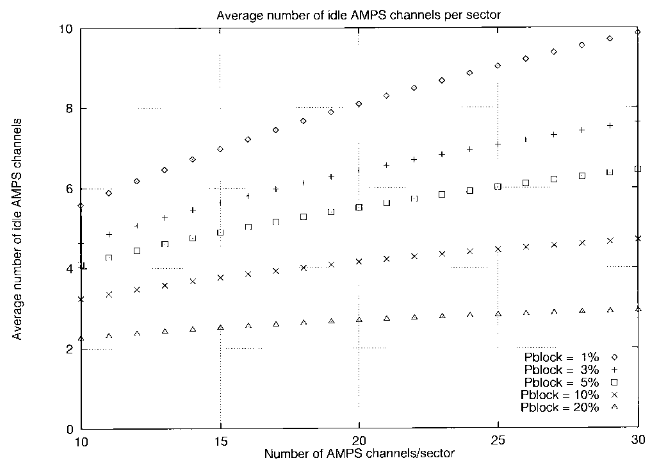

Fig. 2. Average number of idle AMPS channels in a sector.

Typical AMPS sectors in North America are equipped with on the order of 10–15 AMPS channels and typical AMPS channel-holding times, averaged over calls that are answered and those that are not answered,[3] as well as calls that are the results of a handoff are on the order of 90 s. Using this average AMPS channel-holding time, we see from Fig. 1 that an AMPS channel in a sector equipped with ten channels and an AMPS call-blocking probability of 3%, when idle, stays idle for an average of approximately 77 s (0.87 AMPS channel-holding times). From Fig. 2, we see that on average 4.6 channels in this sector are idle.

Numbers such as these are part of the driving forces behind CDPD. There are a compellingly abundant number of idle AMPS channels that could be used to send packet data traffic. And because of the relatively long duration of AMPS channel idle periods, a fair amount of data could be sent before the channel is needed again by an AMPS call.

III. CDPD Essentials

In this section, we present a high-level discussion of the way CDPD detects and uses idle AMPS channels and the

[3]Unanswered calls must be considered in this calculation since they tie up an AMPS channel for on the order of 10 s or more.

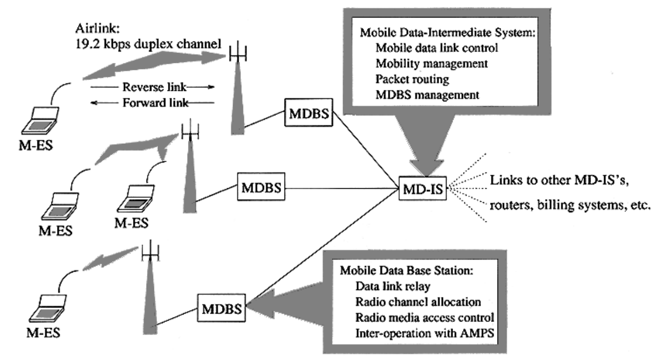

Fig. 3. High-level view of the CDPD network architecture.

CDPD network elements involved. A more detailed summary of CDPD fundamentals can be found in a short tutorial paper [12] or in the CDPD system specification document [1].

Fig. 3 shows the portion of the CDPD network involved in acquiring and using idle AMPS channels. CDPD mobile terminals, known as mobile-end systems (MES's), communicate with the wired portion of the CDPD network via a 19.2-kb/s (raw) duplex wireless link, or *CDPD channel stream*. CDPD channel streams use idle AMPS channels to transmit data to and from MES's. At most, one CDPD channel stream can use an idle AMPS channel at any time. Each AMPS sector may support several CDPD channel streams.

CDPD systems are designed to use idle AMPS channels without direct communication with the AMPS network. To do this, CDPD systems employ a technique called "rf-sniffing" to passively detect whether AMPS channels are idle or busy carrying an AMPS call. Optional provisions are built into the CDPD specification to allow direct communication between CDPD and AMPS so that AMPS can "warn" CDPD of those AMPS channels that will be needed for AMPS calls.

To avoid interference with AMPS calls, CDPD channel streams "hop" from one AMPS channel in the sector to the next, steering clear of AMPS channels carrying AMPS calls or other CDPD channel streams. If an AMPS call suddenly starts using the AMPS channel a CDPD channel stream occupies, the affected CDPD channel stream quickly ceases transmission (in less than 40 ms) and hops to a new idle AMPS channel, if one exists. Cases in which a CDPD channel is preempted by an AMPS call are called *forced hops*. In addition, a CDPD channel stream may be moved to a different AMPS channel in a *planned hop* when preemption by an AMPS call is imminent.

Before moving a CDPD channel stream to a new AMPS channel, the CDPD system attempts to inform the MES's of which AMPS channel the CDPD channel stream plans to hop to. If MES's are informed of the hop, they can quickly find the CDPD channel stream they were last using. Otherwise, MES's lock onto the first acceptable CDPD channel they can find. To assist MES's in reacquiring a CDPD channel stream, a list of AMPS channels that are likely to be used in the event of a channel hop are periodically sent over each channel stream.

If a CDPD channel stream hops and no idle AMPS channel can be found, the channel stream enters a *blackout period* during which no data can be transmitted or received over the CDPD channel stream. A blacked-out CDPD channel stream

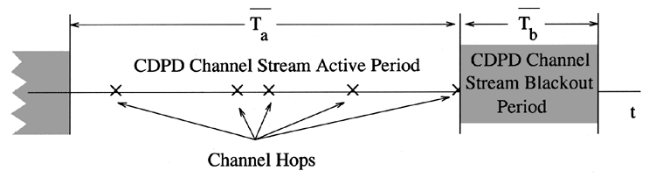

Fig. 4. Active and blackout periods observed by a CDPD channel stream.

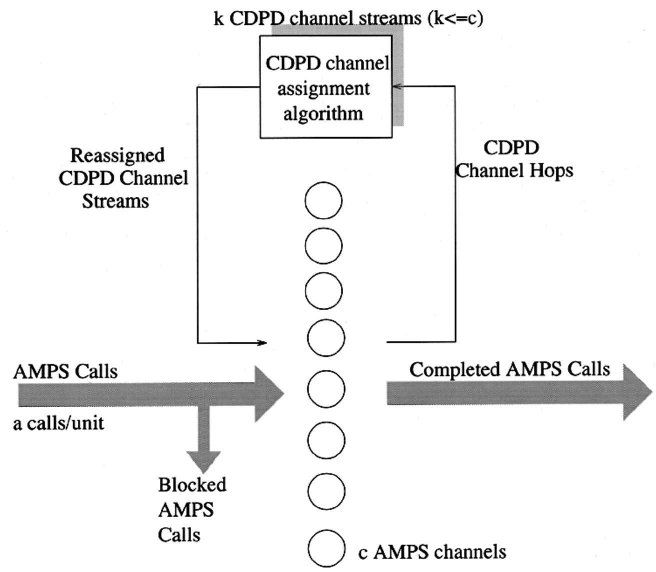

Fig. 5. CDPD channel stream assignment model.

will become active again when the system is able to assign the CDPD channel to an idle AMPS channel. Individual CDPD channel streams alternate between blackout and active periods as shown in Fig. 4. During a CDPD channel stream's active period, none or many planned or forced channel hops may be observed.

To gain access to the CDPD network, MES's lock onto the strongest "acceptable" CDPD channel stream they can find and *register*. The registration process serves two main purposes: 1) it protects the CDPD network against fraudulent use—the system checks the credentials of registering mobiles before granting them access to the CDPD network and 2) it informs the CDPD network of the current network location of the MES. Assuming the CDPD network grants access to the MES, the CDPD network updates a mobility database with the MES's current network location. Once access is granted, the MES continues to listen to the same CDPD channel stream unless the MES or CDPD network decide to initiate a handoff, or the channel stream enters a blackout period.

Both the forward and reverse airlinks of a CDPD channel stream are slotted, digital channels. MES's gain access to the reverse link using a medium-access control protocol similar to the slotted carrier sense multiple access with collision detection scheme used by Ethernet [13]. MES messages sent on the reverse link are broken down into link-layer frames. Frames are formatted and further broken down into small bursts that are encoded and transmitted over the reverse link of the CDPD channel stream. The forward link of the CDPD channel stream, in addition to carrying link-layer frames to the

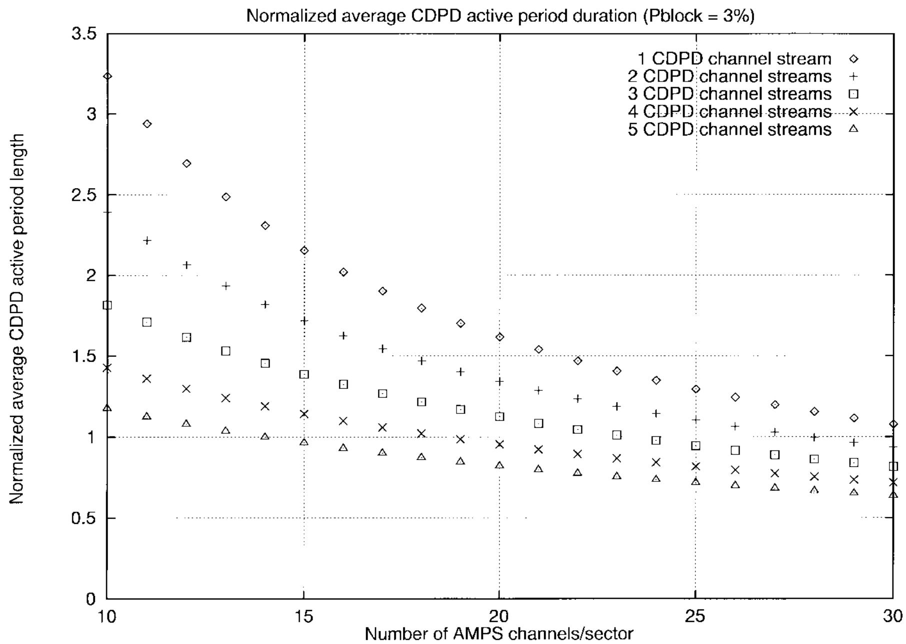

Fig. 6. Mean duration of CDPD channel stream active periods for sectors containing one–five CDPD channel streams. In each case, AMPS call blocking is 3%.

MES's, also carries system overhead and signaling messages, reverse channel busy/idle status, and reverse channel collision feedback.

The CDPD airlink is terminated on the network side at the mobile data base station (MDBS). The MDBS is responsible for finding idle AMPS channels and assigning CDPD channel streams to them. The MDBS relays link-layer frames between the CDPD airlinks and the mobile data-intermediate system (MDIS), the CDPD switching element responsible for managing mobility. Each MES is assigned a "home MDIS," which is responsible for keeping track of where a mobile can be found. Packets bound for an MES are sent to the MES's home MDIS, which then forwards the packets to the MDIS that is currently serving the MES. The serving MDIS reassembles packets sent over the reverse link and routes them appropriately. The MDIS provides connectivity to CDPD systems of other service providers and other wired networks such as the Internet. The MDIS is also networked with billing systems and other CDPD-specific servers.

IV. A Model of AMPS-CDPD Resource Contention

If more than one CDPD channel stream is configured in a sector, CDPD channel streams compete with one another for idle AMPS channels.

To explore the influence that the AMPS call load (a), number of AMPS channels (c), and number of CDPD channel streams (k) have on the duration of CDPD channel stream active and blackout periods and the total amount of idle AMPS airlink capacity used by CDPD in an AMPS sector, we use the simple model shown in Fig. 5. We model the c AMPS channels in a sector as a bank of c parallel servers with no buffer space. AMPS calls arrive to the system according to a Poisson process with rate a calls/unit. For the sake of analytical tractability, we assume AMPS channel-holding times for AMPS calls are exponentially distributed with unit mean. AMPS calls that arrive and find all c AMPS channels occupied by AMPS calls are lost. Otherwise, an arriving AMPS call is assigned to one of the idle AMPS channels at random. Note that since the AMPS system is unaware of which AMPS channels are being used by CDPD channel streams, "idle AMPS channels" include those that are currently occupied by a CDPD channel stream. If an AMPS call is assigned to an AMPS channel that is occupied by a CDPD channel stream, the CDPD channel stream is forced to hop.

The length of time a CDPD channel stream occupies an idle AMPS channel (on the order of seconds) is long relative to the time it takes a CDPD channel stream to hop to a new AMPS channel (on the order of milliseconds as required in [1]). In light of this difference in time scale, we assume that no data-transmission time is lost when a CDPD channel stream hops from one AMPS channel to the next. In the event of a forced hop, the MDBS chooses one of the idle AMPS channels and assigns the CDPD channel stream to it. We assume that all AMPS channels in the sector can be used by CDPD channel streams and that none of the k CDPD channel streams ($k \leq c$) have dedicated use of an AMPS channel. If the CDPD system cannot find an idle AMPS channel (AMPS channels not carrying AMPS calls or other CDPD channel streams), the CDPD channel stream enters a blackout period. Blacked-out channel streams are reassigned to AMPS channels as AMPS channels become idle.

V. How Efficiently Does CDPD Use Idle AMPS Capacity?

Clearly, increasing the number of CDPD channel streams in an AMPS sector does not decrease the amount of idle AMPS air time used by CDPD. If there are more CDPD channel streams, CDPD can use more idle AMPS channels when they are available. Furthermore, the amount of idle airlink capacity in an AMPS sector used by CDPD is maximized when there

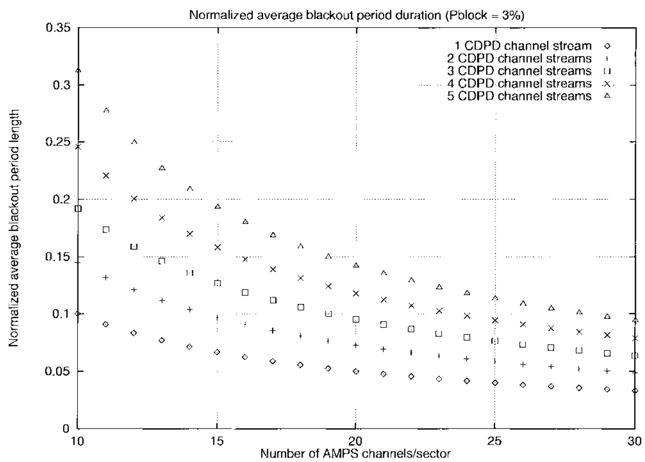

Fig. 7. Mean duration of CDPD channel stream blackout periods for sectors containing one–five CDPD channel streams. In each case, AMPS call blocking is 3%.

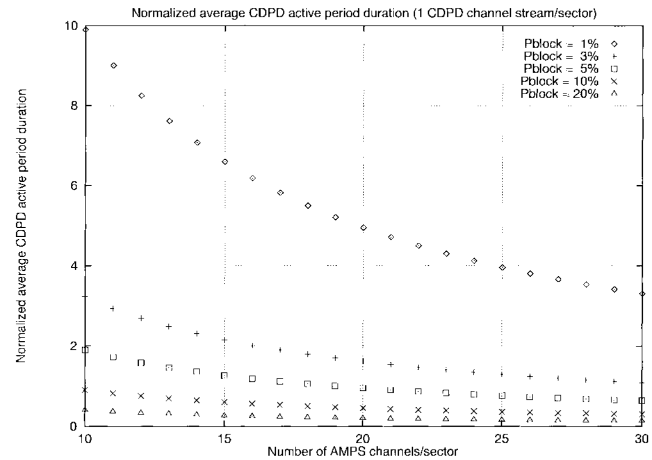

Fig. 8. Mean duration of CDPD channel stream active periods for sectors containing one CDPD channel stream.

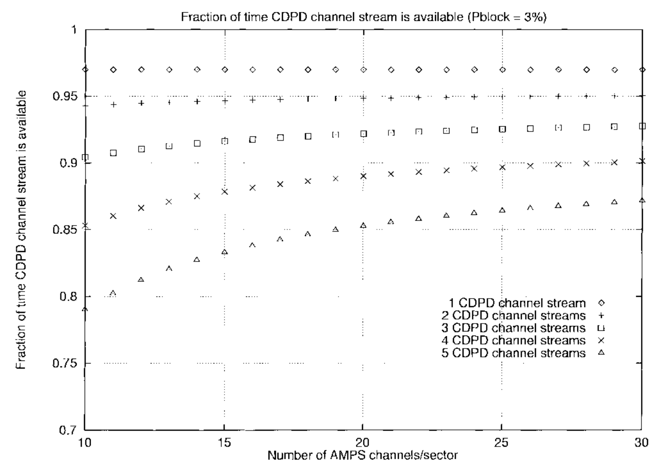

Fig. 9. Mean fraction of time CDPD channel streams are active for sectors containing one–five CDPD channel streams. In each case, AMPS call blocking is 3%.

are as many CDPD channels as there are AMPS channels ($k = c$). As more CDPD channel streams are added to a sector, however, we quickly reach a point of diminishing return.

As more CDPD channels are added to a sector, competition between CDPD channel streams for idle AMPS capacity increases. As a result of this competition, each CDPD channel stream tends to spend more time in the blackout state and less time in the active state.

Consider the model of AMPS-CDPD resource contention introduced in Section IV. Let P_i denote the probability that i AMPS channels are occupied by AMPS calls. P_i is simply the probability that i servers are busy in an M/M/c/c queuing system serving an offered load of a Erlangs [9]

$$P_i = \frac{\frac{a^i}{i!}}{\sum_{j=0}^{c} \frac{a^j}{j!}}. \tag{7}$$

Let $\overline{\lambda}_b$ denote the average rate at which CDPD channel streams are blacked out. We have

$$\begin{aligned}\overline{\lambda}_b &= \sum_{i=0}^{k-1} P_{c-i}(c-i) \\ &= a \sum_{i=c-k}^{c-1} P_i \end{aligned} \tag{8}$$

since the average rate at which channel streams become blacked out is equal to the average rate at which AMPS channels become available to activate blacked-out CDPD channel streams.

Let $\overline{L}_b$ denote the average number of CDPD channel streams that are blacked out at any given time. If i CDPD channel streams are blacked-out, $c - k + i$ AMPS channels must be occupied by AMPS calls. Hence

$$\overline{L}_b = \sum_{i=0}^{k-1} P_{c-i}(k-i). \tag{9}$$

Let $\overline{T}_b$ denote the average length of time a CDPD channel stream is blacked out. Applying Little's Law [9] to a fictitious queue holding blacked-out CDPD channel streams, we find

$$\overline{T}_b = \frac{\overline{L}_b}{\overline{\lambda}_b}. \tag{10}$$

Let $\overline{T}_a$ denote the mean length of time a CDPD channel stream is active. Using arguments similar to those used to derive the expression in (10), we obtain

$$\overline{T}_a = \frac{k - \overline{L}_b}{\overline{\lambda}_b}. \tag{11}$$

To illustrate the effect of adding channel streams to a sector, Figs. 6 and 7 show plots of the average length of a CDPD channel stream's active and blackout periods, respectively, for various-sized AMPS sectors ($p_{\text{block}} = 3\%$). Note that as the number of CDPD channel streams per sector increases, the durations of active periods decrease while the durations of blackout periods increase. In addition, as the number of AMPS channels per sector increases (and the AMPS call-blocking probability and number of CDPD channel streams are

# of CDPD Channels	Fraction of time each CDPD channel stream is active	Mean duration of CDPD channel stream active periods units : mean voice call holding time	Mean duration of CDPD channel stream blackout periods units : mean voice call holding time	Average number of available CDPD channel streams	% of available idle time used by CDPD
1	0.9900	9.900	0.100	0.99	17.73
2	0.9788	6.488	0.141	1.96	35.06
3	0.9600	4.340	0.181	2.88	51.58
4	0.9303	3.023	0.226	3.72	66.65
5	0.8870	2.224	0.283	4.44	79.44
6	0.8297	1.747	0.359	4.98	89.16
7	0.7613	1.473	0.462	5.33	95.45
8	0.6885	1.333	0.603	5.51	98.66
9	0.6191	1.277	0.785	5.57	99.79
10	0.5583	1.264	1.000	5.58	100.00

Fig. 10. CDPD channel stream performance figures for an AMPS sector containing ten AMPS channels and serving an AMPS call load sufficient for an AMPS call-blocking probability of 3%.

held constant), the CDPD channel stream active and blackout periods become shorter.

Fig. 8 shows plots of the mean active-period length for cell sites with one CDPD channel stream as a function of the number of AMPS channels per cell site for several different blocking probabilities. Under our modeling assumption of exponential AMPS channel-holding times, lengths of blackout periods are independent of blocking probability for this configuration. See Fig. 7 for the mean length of blackout periods for this configuration.

Since CDPD channel streams alternate between blackout and active periods, the fraction of time a CDPD channel stream is active equals

$$\frac{\overline{T}_a}{\overline{T}_a + \overline{T}_b}. \tag{12}$$

Fig. 9 shows the fraction of time a CDPD channel stream is active for various configurations when the offered AMPS call load is sufficient to cause a 3% AMPS call-blocking probability. The data-carrying capacity of a CDPD channel stream can be estimated as the data-carrying capacity of a CDPD channel stream on a dedicated channel (no hopping) multiplied by the fraction of time the nondedicated CDPD channel stream is active. The number of CDPD channel streams in a sector multiplied by the fraction of time each is active gives the average number of CDPD channel streams that are active at any given time and, hence, gives us a measure of the CDPD data-carrying capacity of the AMPS sector.

Fig. 10 gives a summary of these CDPD channel stream performance metrics for a sector equipped with ten AMPS channels and an AMPS call-blocking probability of 3% for one–ten CDPD channel streams.

Figures such as those given in Fig. 10 can provide the basis of CDPD capacity engineering rules, allowing service providers to compare the cost of provisioning CDPD channel streams in an AMPS sector with the increase in data-carrying capacity (revenue) per sector.

VI. CDPD Brownout Periods

Due to the stochastic nature of AMPS call attempts and completions, the number of idle AMPS channels is subject to random fluctuations. We refer to periods of time when there are not enough idle AMPS channels to accommodate all k CDPD channel streams in a sector as *CDPD brownout periods.* Periods during which there is an ample number of idle AMPS channels to accommodate all k CDPD channel streams are referred to as *CDPD all channel streams active periods.*

Let $N(t)$ denote the number of AMPS channels that are busy serving AMPS calls at time t. Fig. 11 shows a typical sample path of $N(t)$. When $N(t) \le c-k$, all CDPD channel streams are active. When $N(t) > c-k$, $N(t)-c+k$ CDPD channel streams are blacked out, and the CDPD system is experiencing a brownout.

To study the statistics of the lengths of time all CDPD channel streams in a sector are active and the length of CDPD brownout periods, we continue to use the model introduced in Section IV.

To determine the duration of the periods of time all CDPD channel streams remain active, we determine the probability distribution of the length of the time periods during which $c-k$ or fewer AMPS channels are busy with AMPS calls. This is precisely the problem studied in earlier works in [2] and [15]. In these works, Halfin and Segal explore the issues of using idle trunks in the toll network for the transmission of data—precisely the principle employed by CDPD.

Following [2] and [15], let $f_n(t)$ denote the probability density function of the length of time n or more AMPS channels in the sector are occupied with AMPS calls. Let $F_n(s)$ denote the Laplace transform of $f_n(t)$. As derived in [2], assuming AMPS calls arrive in accordance with a Poisson process of rate a and AMPS channel-holding times are exponentially distributed with the unit mean, $F_n(s)$ has the following simple recursive form:

$$F_n(s) = \begin{cases} \dfrac{n}{a+n+s-F_{n+1}(s)} & 1 \le n < c \\ \dfrac{c}{c+s} & n = c. \end{cases} \tag{13}$$

From (13) we have the following recursive expression for the mean length of time n or more AMPS channels in the

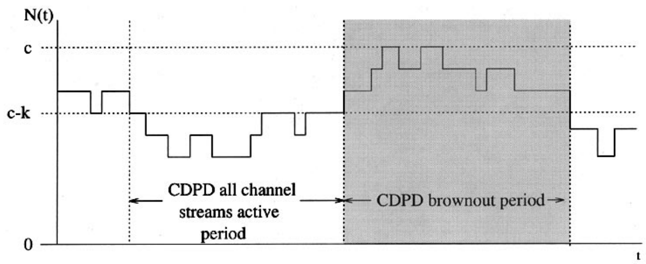

Fig. 11. Sample path showing the relationship between the number of AMPS channels in a sector serving AMPS calls and CDPD brownout periods.

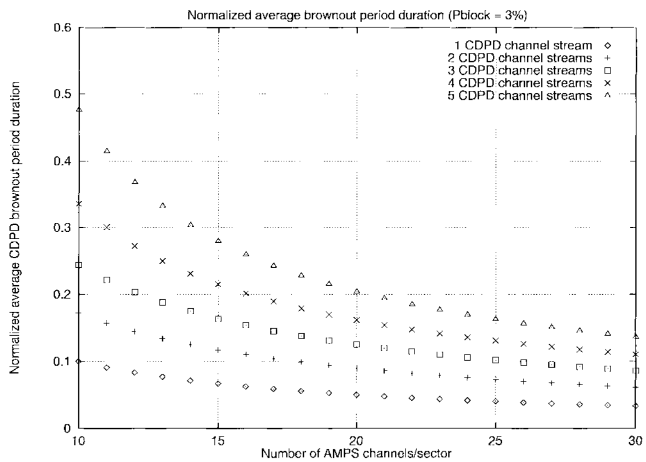

Fig. 12. Mean length of CDPD brownout periods for AMPS sectors with a AMPS call-blocking probability of 3%.

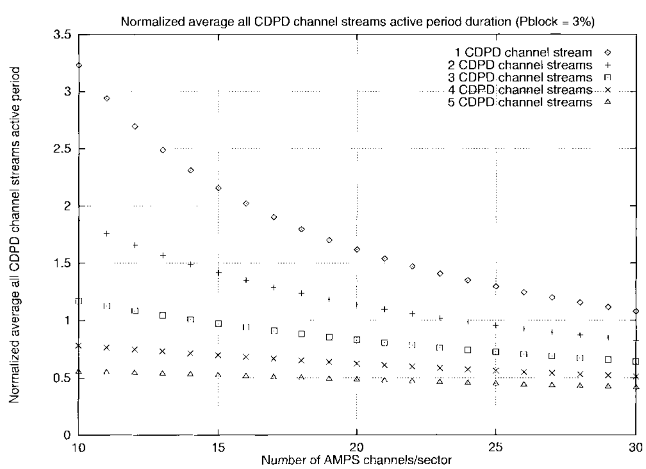

Fig. 13. Mean length of CDPD all channel streams active periods for AMPS sectors with an AMPS call-blocking probability of 3%.

sector are occupied with AMPS calls, $\overline{F}_n$:

$$\overline{F}_n = \begin{cases} \dfrac{1 + a\overline{F}_{n+1}}{n} & 1 \le n < c \\ \dfrac{1}{c} & n = c. \end{cases} \tag{14}$$

Furthermore, let $h_n(t)$ and $H_n(s)$ denote, respectively, the probability density function and its Laplace transform of the

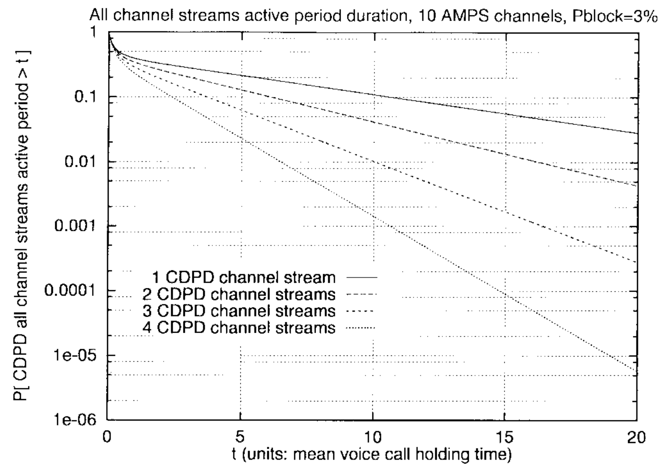

Fig. 14. Complementary distribution function of length of time all CDPD channel streams are active for a sector with ten AMPS channels and an AMPS call-blocking probability of 3%.

length of time fewer than n AMPS channels are occupied with AMPS calls. As derived in [2], $H_n(s)$ can also be expressed recursively:

$$H_n(s) = \begin{cases} \dfrac{a}{a + (n-1) + s - (n-1)H_{n-1}(s)} & 0 < n \le c \\ \dfrac{a}{a+s} & n = 0. \end{cases} \tag{15}$$

From (15) we have the following recursive expression for the mean length of time fewer than n AMPS channels in the sector are occupied with AMPS calls, $\overline{H}_n$:

$$\overline{H}_n = \begin{cases} \dfrac{1 + (n-1)\overline{H}_{n-1}}{a} & 0 < n \le c \\ \dfrac{1}{a} & n = 1. \end{cases} \tag{16}$$

The Laplace transform of the probability density function of the length of the CDPD brownout period is simply $F_{(c-k+1)}(s)$, and its mean length is $\overline{F}_{(c-k+1)}$. Similarly, the Laplace transform of the probability density function of the length of time all CDPD channel streams are active is $H_{(c-k+1)}(s)$, and its mean length is $\overline{H}_{(c-k+1)}$.

Figs. 12 and 13 show the normalized mean duration of CDPD brownout and CDPD all channel streams active periods for AMPS sectors offering a 3% AMPS call-blocking probability as a function of the number of AMPS channels in a sector for different CDPD configurations.

We use a numerical Laplace-transform inversion technique [5] to calculate the complementary distribution functions of the CDPD all channel streams active and CDPD brownout periods. As an example, Figs. 14 and 15 show the complementary distribution functions of the lengths of time all CDPD channel streams are active and the time spent in CDPD brownout for a sector with ten AMPS channels and an AMPS call-blocking probability of 3% (5.529 Erlangs). Fig. 14 shows that in this

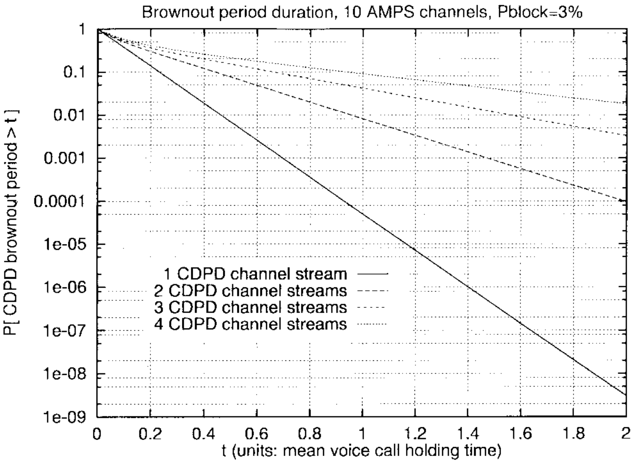

Fig. 15. Complementary distribution function of length of CDPD brownout periods for a sector with ten AMPS channels and an AMPS call-blocking probability of 3%.

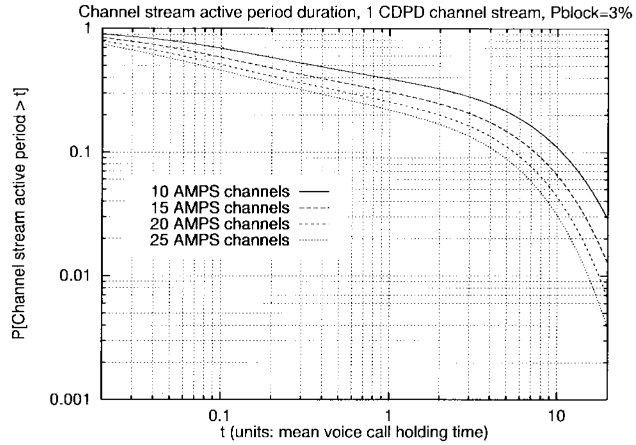

Fig. 17. Complementary distribution function of the length of CDPD channel stream active periods for a sector with one CDPD channel stream and an AMPS call-blocking probability of 3%.

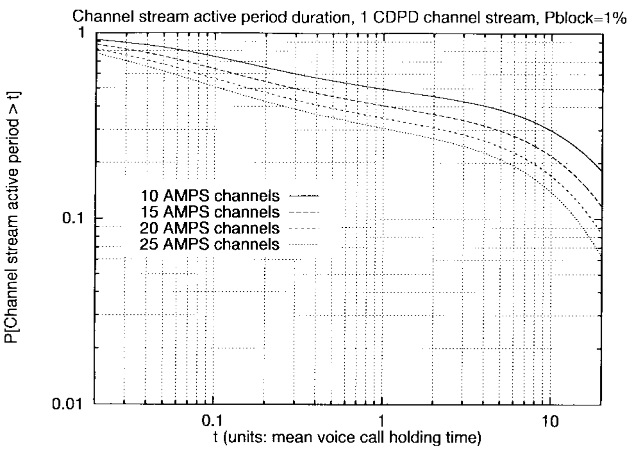

Fig. 16. Complementary distribution function of the length of CDPD channel stream active periods for a sector with one CDPD channel stream and an AMPS call-blocking probability of 1%.

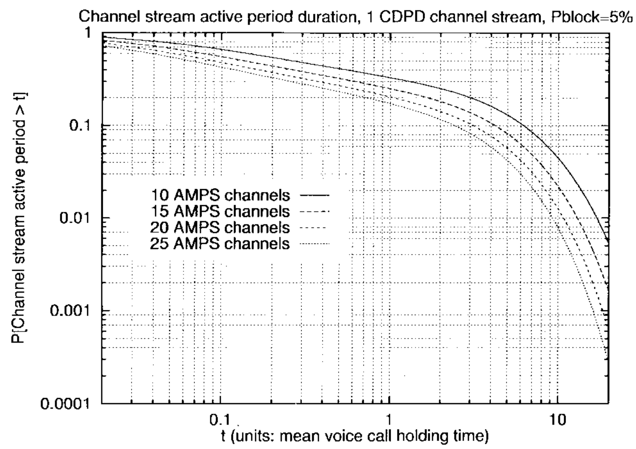

Fig. 18. Complementary distribution function of the length of CDPD channel stream active periods for a sector with one CDPD channel stream and an AMPS call-blocking probability of 5%.

scenario, the distribution functions of the CDPD all channel streams active period have long tails; long periods of time in which all CDPD channel streams are active occur fairly frequently.

A useful application of this analysis is the case in which only one CDPD channel stream is assigned to a sector. In this configuration, the distribution of the all CDPD channel streams active period is equivalent to the distribution of the channel stream's active period. Likewise, the distribution of the CDPD brownout period is equivalent to the channel stream's blackout period.

Figs. 16–20 demonstrate the effect of the AMPS call load and number of AMPS channels per sector on the distribution of the duration of a CDPD channel stream's active period for sectors configured with one CDPD channel stream (mean duration of active periods for this configuration is shown in Fig. 8).

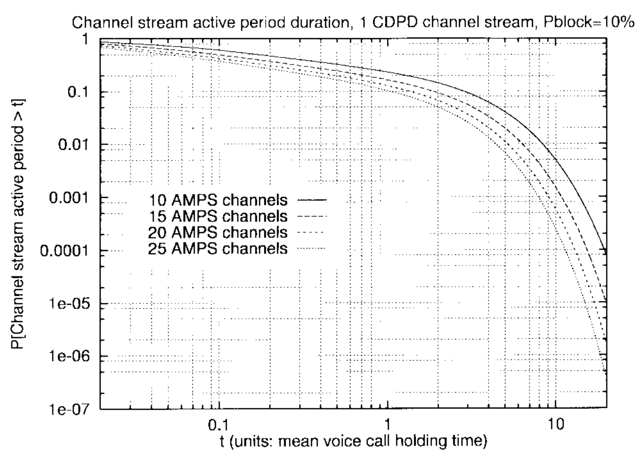

Fig. 19. Complementary distribution function of the length of CDPD channel stream active periods for a sector with one CDPD channel stream and an AMPS call-blocking probability of 10%.

Fig. 16 shows that in a sector with ten AMPS channels, roughly 50% of the CDPD channel stream's active periods last longer than one mean AMPS channel-holding time. With 25 AMPS channels in a sector, the fraction of CDPD channel stream active periods lasting longer that one mean AMPS channel-holding time drops to 30%.

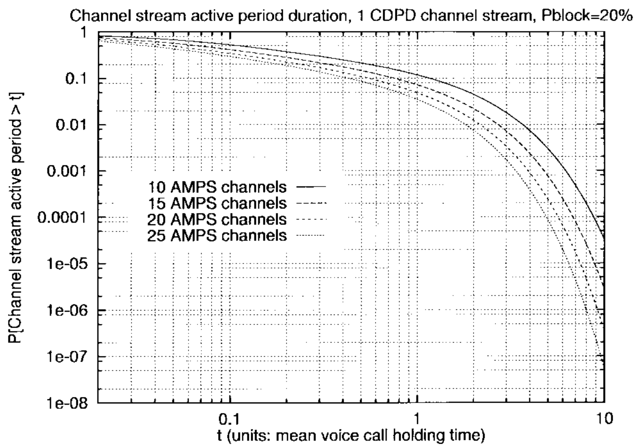

Fig. 20. Complementary distribution function of the length of CDPD channel stream active periods for a sector with one CDPD channel stream and an AMPS call-blocking probability of 20%.

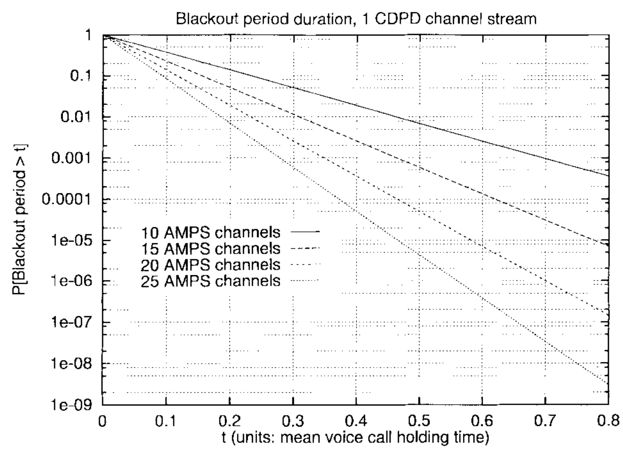

Fig. 21. Complementary distribution function of the length of time a CDPD channel is blacked out for sectors with one CDPD channel stream. The blackout period for this configuration is independent of AMPS call load.

Comparing Figs. 17 and 19, for example, we see the impact of AMPS call load on the duration of CDPD channel stream active periods. In a sector with ten AMPS channels and an AMPS blocking rate of 3%, 39% of the channel stream's active periods last longer than one mean AMPS channel-holding time compared to only 23% at an AMPS blocking rate of 10%.

At like AMPS call-blocking probabilities, a CDPD channel stream's active periods are shorter in sectors with a larger number of AMPS channels. The lengths of blackout periods are also shorter in sectors with larger numbers of AMPS channels. Under our modeling assumptions, a channel stream's blackout period in a sector with c AMPS channels and one CDPD channel stream is exponentially distributed with mean c^{-1}. The complementary distribution function of the duration of blackout periods is shown in Fig. 21 (mean duration of blackout periods in a sector with one CDPD channel stream is independent of AMPS call load and is given in Fig. 12).

Fig. 21 shows that the tails of the blackout periods are relatively short (a result of our assumption that AMPS channel-holding times are exponentially distributed). For example, in an AMPS sector with ten AMPS channels and one CDPD channel stream, 90% of the channel stream blackout periods last less than 0.23 mean AMPS channel-holding times. In an AMPS sector with 25 AMPS channels, over 90% of the channel-stream blackout periods last less than 0.1 mean AMPS channel-holding times. Keeping blackout periods short is important for good delay performance. The implicit assumption in deploying CDPD on nondedicated AMPS channels is that the data carried is relatively insensitive to delay. If blackout periods are too long (or active periods are too short), it may be necessary to deploy the CDPD channel stream on a dedicated AMPS channel.

VII. Summary

From the analysis presented in this paper we draw the following broad conclusions.

1) Due to trunking inefficiencies inherent to AMPS systems, there is a relatively large amount of spare AMPS airlink capacity that can be used to carry CDPD data traffic. The total amount of spare capacity available to CDPD in an AMPS sector can be easily calculated from the offered AMPS call load and the number of AMPS channels equipped in the sector.
2) The amount of spare AMPS capacity available to CDPD is independent of the mean AMPS channel-holding time. At like AMPS call-blocking probabilities, however, the periods of time AMPS channels are idle will be shorter in AMPS systems with shorter mean AMPS channel-holding times. Shorter AMPS channel-idle periods will force CDPD channel streams to hop more frequently.
3) At like AMPS call-blocking probabilities and mean AMPS channel-holding times, the length of time an AMPS channel is idle decreases as the number of AMPS channels in an AMPS sector increases. Under these conditions, as the number of AMPS channels per sector grows, CDPD channel streams will be forced to hop more frequently.
4) As the AMPS call load in an AMPS sector increases, the amount of airlink capacity available to CDPD decreases. Also, the length of time each AMPS channel is idle decreases, causing CDPD channel streams to hop more frequently.
5) Increasing the number of CDPD channel streams equipped in an AMPS sector increases the amount of data that can be carried by CDPD. At peak call loads, however, we quickly reach a point of diminishing return: adding new CDPD channel streams does not buy all that much extra CDPD data-carrying capacity. Results presented in this paper can be used to quantify the CDPD data-carrying capacity of an AMPS sector equipped with an arbitrary number of CDPD channel streams.

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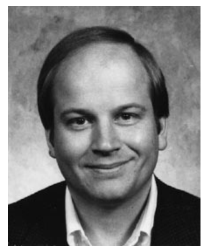

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